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QUADRATURE ANALYSIS OF ELASTICITY SUPPORTED PIEZOELECTRIC NANO BEAM

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Abstract

This work concerns with free vibration analysis of piezoelectric Nano beam resting on nonlinear Winkler –Pasternak foundation. Based on Hamilton principle, governing equations of the problem are derived. Three differential quadrature techniques are employed to reduce the problem to an Eigen-value problem. The obtained nonlinear algebraic system is solved by using iterative quadrature technique. That is solved for different materials and boundary conditions. The natural frequencies of Nano elastic beam based on nonlocal elasticity theory and Timoshenko beam theory are obtained. Numerical analysis is introduced to explain influence of computational characteristics of the proposed schemes on convergence, accuracy and efficiency of the obtained results. The obtained results agreed with the previous analytical and numerical ones. Furthermore, a parametric study is introduced to investigate the influence of linear and nonlinear elastic foundation, temperature change, external electric voltage, nonlocal parameter and length-to-thickness ratio on the values of natural frequencies and mode shapes of the piezoelectric Nano beam.

Keywords: Nonlocal Elasticity Theory; Timoshenko Theory; Vibration; Piezoelectric; Nonlinear Elastic Foundation; SINC; Discrete Singular Convolution.

1. Introduction

Piezoelectric Nano elastic beams have been found a wide range of applications especially for automobile, aircrafts, electronic, biomedical sectors and several engineering structures [1-5]. Vibration analysis of such Nano elastic beams play vital role in various engineering applications as structural component. Nanostructures based on piezoelectric Nano materials have been found a great attention from research communities [6-7].

Due to the complexity of such problems, only limited cases can be solved analytically [8-11]. Finite elements [12], meshless [13], Galerkin [14], spline finite strip [15], least squares [16] and Rayleigh-Ritz [17] techniques have been used to solve such Nano elastic beam problems. The disadvantages of these numerical methods are the need to large number of grid points. Also, these methods need a large computational time to obtain the accuracy.

Lately, a differential quadrature method (DQM) is the most popular method in the numerical solutions of boundary value problems [18]. This method leads to accurate solutions with fewer grid points. The convergence and stability of this method depending on choice of shape function. It used to determine the weighting coefficients. Lagrange interpolation polynomials, Cardinal sine function, Delta Lagrange Kernel (DLK) and Regularized Shannon kernel (RSK) are shape functions which by termed polynomial based DQM (PDQM), Sinc differential quadrature method (SDQM) [19], and Discrete singular convolution differential quadrature method (DSCDQM) [20]. SDQM and DSCDQM are more reliable versions than polynomial based DQM.

Up to knowledge of the authors, SDQM and DSCDQM are not examined for vibration analysis of piezoelectric Nano elastic beam. Also, the present work extends the applications of DQM to analyze this problem. Based on these versions, numerical schemes are designed for free vibration of piezoelectric Nano elastic beams. A matlab program is designed to solve this problem. The natural frequencies are obtained and compared with previous analytical and numerical ones. For each scheme the convergence and efficiency is verified. Also, a parametric study is introduced to investigate the influence of linear and nonlinear elastic foundation, temperature change, external electric voltage, nonlocal parameter and length-to-thickness ratio on the values of natural frequencies and mode shapes.

2. Formulation of the Problem

Consider a piezoelectric Nano elastic beam with $(0 \le x \le L, \quad 0 \le z \le h)$ where L and h are length and thickness of the beam. This beam is polarized in z direction and subjected to an applied voltage $\phi(x, z, t)$, a uniform temperature change ΔT and linear and nonlinear elastic foundation K_1, K_2, K_3 respectively as shown in Fig.(1).

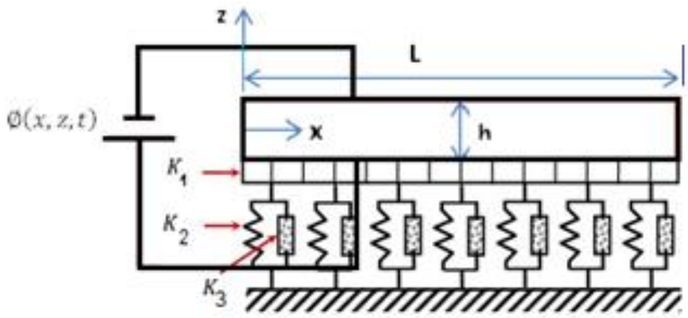

Fig.1. Piezoelectric Nano elastic

Based on Hamilton principle, equations of motion can be written as [21]:

$$A_{11}\frac{\partial^2 U}{\partial x^2} = I_1\frac{\partial^2}{\partial t^2}\left[U-(e_0a)^2\frac{\partial^2 U}{\partial x^2}\right] \quad (1)$$

$$k_sA_{44}\left[\frac{\partial^2 W}{\partial x^2}+\frac{\partial \Psi}{\partial x}\right]-k_sE_{15}\frac{\partial^2\phi}{\partial x^2}+(N_E+N_T)\frac{\partial^2 W}{\partial x^2}-(N_E+N_T)(e_0a)^2\frac{\partial^4 W}{\partial x^4}+K_1W-K_2\frac{\partial^2 W}{\partial x^2}+K_3W^3$$
$$=I_1\frac{\partial^2}{\partial t^2}\left[W-(e_0a)^2\frac{\partial^2 W}{\partial x^2}\right], \quad (2)$$

$$D_{11}\frac{\partial^2\Psi}{\partial x^2}-k_sA_{44}\left(\frac{\partial W}{\partial x}+\Psi\right)+F_{31}\frac{\partial\phi}{\partial x}+k_sE_{15}\frac{\partial\phi}{\partial x}=I_3\frac{\partial^2}{\partial t^2}\left[\Psi-(e_0a)^2\frac{\partial^2\Psi}{\partial x^2}\right], \quad (3)$$

$$F_{31}\frac{\partial\Psi}{\partial x}+E_{15}\left[\frac{\partial^2 W}{\partial x^2}+\frac{\partial\Psi}{\partial x}\right]+X_{11}\frac{\partial^2\phi}{\partial x^2}-X_{33}\phi=0, \quad (4)$$

Where U, W and Ψ are displacement in mid-plane and cross section rotation respectively; (e_0a) is the scale coefficient which incorporates the small scale effect. k_s is the shear correction factor which takes values $5/6$ for the macro scale beams [21].

$$N_T=-\lambda_1 h\Delta T \;,\; N_E=2e_{31}V_0 \quad (5)$$

Where N_T, N_E are normal force induced by the temperature change ΔT and normal force induced by the external electric voltage V_0. thermal module and piezoelectric are λ_1, e_{31} constant. K_1, K_2, K_3 are shear and spring coefficients of linear elastic foundation and nonlinear elastic foundation [22].

$$
\left(A_{11},\ A_{44}\right)=\left(C_{11},C_{44}\right)h,\ D_{11}=\ C_{11}h^{3}/12,\ E_{15}=2\frac{e_{15}}{\beta}\sin\left(\frac{\beta h}{2}\right),\left(I_{1},I_{3}\right)=\rho\left(h,h^{3}/12\right),
$$

$$
F_{31}=\ e_{31}\left[-h\cos\left(\frac{\beta h}{2}\right)+\frac{2}{\beta}\sin\left(\frac{\beta h}{2}\right)\right],X_{11}=\ \frac{\epsilon_{11}}{2}\left[h+\frac{\sin(\beta h)}{\beta}\right],\ X_{33}=\ \frac{\epsilon_{33}\beta^{2}}{2}\left[h-\frac{\sin(\beta h)}{\beta}\right],\beta=\pi/h \tag{6}
$$

Where D, E, C, e, ρ *and* ϵ are electric field, electric displacement, elastic constant, piezoelectric constant, mass density and dielectric constants.

The field quanties are normalized such as :

$$
\zeta=\frac{x}{L},\ w=\frac{W}{h},\ \psi=\Psi,\ \eta=\frac{L}{h},\ \mu=\frac{e_{0}a}{L},\ \varphi=\frac{\phi}{\phi_{0}},\ \phi_{0}=\sqrt{\frac{\varepsilon_{33}}{A_{11}}},\ \bar{A}_{11}=\frac{A_{11}}{A_{11}},\ \bar{A}_{44}=\frac{A_{44}}{A_{11}},
$$

$$
\bar{D}_{11}=\frac{D_{11}}{A_{11}h^{2}},\ \bar{I}_{1}=\frac{I_{1}}{I_{1}},\ \bar{I}_{3}=\frac{I_{3}}{I_{1}h^{2}},\ \bar{X}_{11}=\frac{X_{11}\phi_{0}^{2}}{A_{11}h^{2}},\ \bar{X}_{33}=\frac{X_{33}\phi_{0}^{2}}{A_{11}},\ \bar{E}_{15}=\frac{E_{15}\phi_{0}}{A_{11}h},\ \bar{F}_{31}=\frac{F_{31}\phi_{0}}{A_{11}h} \tag{7}
$$

$$
\bar{N}_{T}=-\frac{\lambda h\,\Delta T}{A_{11}},\ \ \bar{N}_{E}=\frac{2e_{31}V_{0}}{A_{11}},\ \ \tau=\frac{t}{L}\sqrt{\frac{I_{1}}{A_{11}}},\ \ k_{1}=\frac{K_{1}L^{4}}{\pi^{2}A_{11}h^{2}},\ k_{2}=\frac{K_{2}L^{2}}{\pi^{2}A_{11}h^{2}},\ k_{3}=\frac{K_{3}L^{2}}{\pi^{2}A_{11}h^{2}},
$$

Further, for harmonic behavior, one can assume that:

$$
U(x,t)=u\,e^{i\omega t},\quad W(x,t)=w\,e^{i\omega t},\ \ \Psi(x,t)=\psi e^{i\omega t},\ \ \phi(x,t)=\varphi\,e^{i\omega t} \tag{8}
$$

where ω is the natural frequency of the beam and $i=\sqrt{-1}$.

u,w,ψ and ϕ ,are the amplitudes for U,W,Ψ *and* Φ respectively.

Substituting from equations (7-8) into (1-4), the problem can be reduced to a quasi-static one as:

$$
\bar{A}_{11}\frac{\partial^{2}u}{\partial\zeta^{2}}=-\omega^{2}\bar{I}_{1}\left[u-\mu^{2}\frac{\partial^{2}u}{\partial\zeta^{2}}\right], \tag{9}
$$

$$
\begin{aligned}
&k_{s}\bar{A}_{44}\left[\frac{\partial^{2}w}{\partial\zeta^{2}}+\eta\frac{\partial\psi}{\partial\zeta}\right]-k_{s}\bar{E}_{15}\frac{\partial^{2}\varphi}{\partial\zeta^{2}}+\left(\bar{N}_{E}+\bar{N}_{T}\right)\frac{\partial^{2}w}{\partial\zeta^{2}}-\left(\bar{N}_{E}+\bar{N}_{T}\right)\mu^{2}\frac{\partial^{4}w}{\partial\zeta^{4}}+\\
&k_{1}w-k_{2}\frac{\partial^{2}w}{\partial\zeta^{2}}+k_{3}w^{3}=-\omega^{2}\bar{I}_{1}\left[w-\mu^{2}\frac{\partial^{2}w}{\partial\zeta^{2}}\right],
\end{aligned} \tag{10}
$$

$$
\bar{D}_{11}\frac{\partial^{2}\psi}{\partial\zeta^{2}}-k_{s}\bar{A}_{44}\eta\left(\frac{\partial w}{\partial\zeta}+\eta\psi\right)+(\bar{F}_{31}+k_{s}\bar{E}_{15})\eta\frac{\partial\varphi}{\partial\zeta}=-\omega^{2}\bar{I}_{3}\left[\psi-\mu^{2}\frac{\partial^{2}\varphi}{\partial\zeta^{2}}\right], \tag{11}
$$

$$
\bar{F}_{31}\eta\frac{\partial\psi}{\partial\zeta}+\bar{E}_{15}\left[\frac{\partial^{2}w}{\partial\zeta^{2}}+\eta\frac{\partial\psi}{\partial\zeta}\right]+\bar{X}_{11}\frac{\partial^{2}\varphi}{\partial\zeta^{2}}-\bar{X}_{33}\eta^{2}\varphi=0 \tag{12}
$$

(1) For Clamped end (C)

$u = w = \psi = \varphi = 0,$

(13)

(2) For Hinged end (H):

$u = w = \varphi = 0,$

$$\bar{D}_{11}\frac{\partial\psi}{\partial\zeta} + \bar{F}_{31}\eta\varphi - \omega^2\mu^2\left[\bar{I}_3\frac{\partial\psi}{\partial\zeta} + \bar{I}_1\eta w - \mu^2\bar{I}_1\eta\frac{\partial^2 u}{\partial\zeta^2}\right] - \eta\left(\bar{N}_E + \bar{N}_T\right)\mu^2\frac{\partial^2 w}{\partial\zeta^2} = 0, \quad (14)$$

The boundary conditions can be described as:

3. **Method of Solution**

Three differential quadrature techniques are employed to reduce the governing equation into an Eigen value problem as follows [18-20]:

- **Polynomial based differential quadrature method (PDQM)**

In this technique, Lagrange interpolation polynomial is employed as a shape function such that the unknown *v* and its derivatives can be approximated as a weighted linear sum of nodal values, v_i , ($i=1:N$), as follows [23]:

$$v(x_i) = \sum_{j=1}^{N} \frac{\prod_{k=1}^{N}(x_i - x_k)}{(x_i - x_j)\prod_{j=1, j\neq k}^{N}(x_j - x_k)} v(x_j), \quad (i = 1:N), \quad (15)$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=x_i} = \sum_{j=1}^{N} C_{ij}^{(1)} v(x_j), \qquad \left.\frac{\partial^2 v}{\partial x^2}\right|_{x=x_i} = \sum_{j=1}^{N} C_{ij}^{(2)} v(x_j), \quad (i = 1:N) \quad (16)$$

Similarly, one can approximate $\frac{\partial^3 v}{\partial x^3}$, $\frac{\partial^4 v}{\partial x^4}$ and calculate $C_{ij}^{(3)}, C_{ij}^{(4)}$

Where *v* terms to u, w, ψ and φ. *N* is the number of grid points. The weighting coefficients $C_{ij}^{(1)}$ can be determined by differentiating (14) as [23]:

$$C_{ij}^{(1)} = \begin{cases} \dfrac{1}{(x_i - x_j)} \displaystyle\prod_{k=1, k\neq i,j}^{N} \dfrac{(x_i - x_k)}{(x_j - x_k)} & i \neq j \\[2ex] -\displaystyle\sum_{j=1, j\neq i}^{N} C_{ij}^{(1)} & i = j \end{cases} \quad (17)$$

Also, by using matrix multiplication can be calculated $C_{ij}^{(2)}, C_{ij}^{(3)}$ *and* $C_{ij}^{(4)}$ as:

$$\left[C_{ij}^{(n)}\right]=\left[C_{ij}^{(1)}\right]\left[C_{ij}^{(n-1)}\right] \quad ,(n=2,3,4) \tag{18}$$

- **Sinc Differential Quadrature Method (SDQM)**

Cardinal sine function is used as a shape function such that the unknown *v* and its derivatives can be approximated as a weighted linear sum of nodal values, v_i, (i= -N: N), as follows:

$$S_j(x_i,h_x)=\frac{\sin[\pi(x_i-x_j)/h_x]}{\pi(x_i-x_j)/h_x}, \text{ Where } (h_x>0) \text{ is the step size.} \tag{19}$$

This function is applied as a shape function such that the unknown *v* and its derivatives are approximated as a weighted linear sum of nodal values, v_i,(i= -N : N), as follows [19]:

$$v(x_i)=\sum_{j=-N}^{N}\frac{\sin[\pi(x_i-x_j)/h_x]}{\pi(x_i-x_j)/h_x}v(x_j), \quad (i=-N:N), h_x > 0 \tag{20}$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=x_i}=\sum_{j=-N}^{N}C_{ij}^{(1)}v(x_j), \quad \left.\frac{\partial^2 v}{\partial x^2}\right|_{x=x_i}=\sum_{j=-N}^{N}C_{ij}^{(2)}v(x_j),$$
$$\left.\frac{\partial^3 v}{\partial x^3}\right|_{x=x_i}=\sum_{j=-N}^{N}C_{ij}^{(3)}v(x_j), \quad \left.\frac{\partial^4 v}{\partial x^4}\right|_{x=x_i}=\sum_{j=-N}^{N}C_{ij}^{(4)}v(x_j), \quad (i=-N:N), \tag{21}$$

Where *v* terms to u, w, ψ and φ. *N* is the number of grid points. h_x is grid size. The weighting coefficients $C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}$ *and* $C_{ij}^{(4)}$ can be determined by differentiating (19) and (20) as:

$$C_{ij}^{(1)}=\begin{cases}\dfrac{(-1)^{i-j}}{h_x\,(i-j)}, & i\neq j\\ 0 & i=j\end{cases}\quad,\quad C_{ij}^{(2)}=\begin{cases}\dfrac{2(-1)^{i-j+1}}{h_x{}^2\,(i-j)^2}, & i\neq j\\ -\dfrac{\pi^2}{3h_x{}^2} & i=j\end{cases}$$

$$C_{ij}^{(3)}=\begin{cases}\dfrac{(-1)^{i-j}}{h_x{}^3(i-j)^3}\left(6-\pi^2(i-j)^2\right), & i\neq j\\ 0 & i=j\end{cases}\quad,C_{ij}^{(4)}=\begin{cases}\dfrac{4(-1)^{i-j+1}}{h_x{}^4(i-j)^4}\left(6-\pi^2(i-j)^2\right), & i\neq j\\ \dfrac{\pi^4}{5h_x{}^4} & i=j\end{cases} \tag{22}$$

- **Discrete Singular Convolution Differential Quadrature Method (DSCDQM)**

A singular convolution can be defined as [20]

$$F_{(t)}=(T*\eta)(t)=\int_{-\infty}^{\infty}T(t-x)\eta(x)dx \tag{23}$$

Where $T(t-x)$ is a singular kernel.

The DSC algorithm can be applied using many types of kernels. These kernels are applied as shape functions such that the unknown *v* and its derivatives are approximated as a weighted linear sum of v_i, (i= $-N$: N), over a narrow bandwidth ($x-x_M, x+x_M$) [24-25].

Two kernels of DSC will be employed as follows:

(a) Delta Lagrange Kernel (DLK) can be used as a shape function such that the unknown *v* and its derivatives can be approximated as a weighted linear sum of nodal values, v_i,(i= $-N$: N), as follows :

$$v(x_i)=\sum_{j=-M}^{M}\frac{\prod_{k=-M}^{M}(x_i-x_k)}{(x_i-x_j)\prod_{j=-M,\,j\neq k}^{M}(x_j-x_k)}v(x_j),\quad (i=-N:N),\,M\geq 1 \tag{24}$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(1)}v(x_j),\qquad \left.\frac{\partial^2 v}{\partial x^2}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(2)}v(x_j),$$
$$\left.\frac{\partial^3 v}{\partial x^3}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(3)}v(x_j),\quad \left.\frac{\partial^4 v}{\partial x^4}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(4)}v(x_j),\quad (i=-N:N), \tag{25}$$

Where 2M+1 is the effective computational band width.

$C_{ij}^{(1)},C_{ij}^{(2)},C_{ij}^{(3)}$ *and* $C_{ij}^{(4)}$ *are defined as* :

$$C_{ij}^{(1)} = \begin{cases} \dfrac{1}{(x_i - x_j)} \displaystyle\prod_{k=-M, k\neq i,j}^{M} \dfrac{(x_i - x_k)}{(x_j - x_k)} & i \neq j \\ -\displaystyle\sum_{j=-M, j\neq i}^{M} C_{ij}^{(1)} & i = j \end{cases}, \tag{26}$$

$$C_{ij}^{(2)} = \begin{cases} 2\left(C_{ij}^{(1)} C_{ii}^{(1)} - \dfrac{C_{ij}^{(1)}}{(x_i - x_j)} \right) & i \neq j \\ -\displaystyle\sum_{j=-M, j\neq i}^{M} C_{ij}^{(2)} & i = j \end{cases}, \tag{27}$$

$$C_{ij}^{(3)} = \begin{cases} 3\left(C_{ij}^{(1)} C_{ii}^{(2)} - \dfrac{C_{ij}^{(2)}}{(x_i - x_j)} \right) & i \neq j \\ -\displaystyle\sum_{j=-M, j\neq i}^{M} C_{ij}^{(3)} & i = j \end{cases}, \tag{28}$$

$$C_{ij}^{(4)} = \begin{cases} 4\left(C_{ij}^{(1)} C_{ii}^{(3)} - \dfrac{C_{ij}^{(3)}}{(x_i - x_j)} \right) & i \neq j \\ -\displaystyle\sum_{j=-M, j\neq i}^{M} C_{ij}^{(4)} & i = j \end{cases}, \tag{29}$$

(b) Regularized Shannon kernel (RSK) can also be used as a shape function such that

(c) the unknown v and its derivatives can be approximated as a weighted linear sum of nodal values v_i, (i=-N:N) as follows :

$$\psi(x_i) = \sum_{j=-M}^{M} \left\langle \frac{\sin[\pi(x_i - x_j)/h_x]}{\pi(x_i - x_j)/h_x} e^{-(\frac{(x_i - x_j)^2}{2\sigma^2})} \right\rangle \psi(x_j), \quad (i = -N : N), \sigma = (r * h_x) > 0 \tag{30}$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(1)}v(x_j),\quad \left.\frac{\partial^2 v}{\partial x^2}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(2)}v(x_j),$$
$$\left.\frac{\partial^3 v}{\partial x^3}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(3)}v(x_j),\ \left.\frac{\partial^4 v}{\partial x^4}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{(4)}v(x_j),\quad (i=-N,N), \tag{31}$$

Where σ is regularization parameter and r is a computational parameter. The weighting coefficients $C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}$ *and* $C_{ij}^{(4)}$ can be defined as [26]:

$$C_{ij}^{(1)}=\begin{cases}\dfrac{(-1)^{i-j}}{h_x(i-j)}e^{-h_x^2(\frac{(i-j)^2}{2\sigma^2})}, & i\neq j\\ 0 & i=j\end{cases},\quad C_{ij}^{(2)}=\begin{cases}\left(\dfrac{2(-1)^{i-j+1}}{h_x^2(i-j)^2}+\dfrac{1}{\sigma^2}\right)e^{-h_x^2(\frac{(i-j)^2}{2\sigma^2})}, & i\neq j\\ -\dfrac{1}{\sigma^2}-\dfrac{\pi^2}{3h_x^2} & i=j\end{cases}$$

$$C_{ij}^{(3)}=\begin{cases}\dfrac{(-1)^{i-j}}{h_x^3(i-j)^3}\left(\dfrac{\pi^2}{h_x^3(i-j)}+\dfrac{6}{h_x^3(i-j)^3}+\dfrac{3}{h_x(i-j)\sigma^2}+\dfrac{3h_x(i-j)}{\sigma^4}\right)e^{-h_x^2(\frac{(i-j)^2}{2\sigma^2})}, & i\neq j\\ 0 & i=j\end{cases} \tag{32}$$

$$C_{ij}^{(4)}=\begin{cases}(-1)^{i-j}\left(\dfrac{4\pi^2}{h_x^4(i-j)^2}+\dfrac{4\pi^2}{h_x^2\sigma^2}-\dfrac{24}{h_x^4(i-j)^4}-\dfrac{12}{h_x^2(i-j)^2\sigma^2}-\dfrac{4h_x^2(i-j)^2}{\sigma^6}\right)e^{-h_x^2(\frac{(i-j)^2}{2\sigma^2})}, & i\neq j\\ \dfrac{3}{\sigma^4}+\dfrac{2\pi^2}{h_x^2\sigma^2}+\dfrac{\pi^4}{5h_x^4} & i=j\end{cases}$$

On suitable substitution from equations of weighting coefficients (32) into (9-12), the problem can be reduced to the following Eigen-value problem:

$$\bar{A}_{11}\sum_{j=1}^{N}C_{ij}^{(2)}u_j=-\bar{I}_1\,\omega^2\left[\sum_{j=1}^{N}\delta_{ij}u_j-\mu^2\sum_{j=1}^{N}C_{ij}^{(2)}u_j\right], \tag{33}$$

$$\mathrm{k_s}\,\bar{A}_{44}\left[\sum_{j=1}^{N}C_{ij}^{(2)}w_j+\eta\sum_{j=1}^{N}C_{ij}^{(1)}\psi_j\right]-\mathrm{k_s}\,\bar{E}_{15}\sum_{j=1}^{N}C_{ij}^{(2)}\varphi_j+\left(\bar{N}_E+\bar{N}_T\right)\sum_{j=1}^{N}C_{ij}^{(2)}w_j-\left(\bar{N}_E+\bar{N}_T\right)\mu^2\sum_{j=1}^{N}C_{ij}^{(4)}w_j+$$
$$k_1\sum_{j=1}^{N}\delta_{ij}w_j-k_2\sum_{j=1}^{N}C_{ij}^{(2)}w_j+k_3\sum_{j=1}^{N}\delta_{ij}w_j^3=-\bar{I}_1\,\omega^2\left[\sum_{j=1}^{N}\delta_{ij}w_j-\mu^2\sum_{j=1}^{N}C_{ij}^{(2)}w_j\right], \tag{34}$$

$$\bar{D}_{11}\sum_{j=1}^{N}C_{ij}^{(2)}\psi_j-\mathrm{k_s}\,\bar{A}_{44}\,\eta\left(\sum_{j=1}^{N}C_{ij}^{(1)}w_j+\eta\sum_{j=1}^{N}\delta_{ij}\psi_j\right)+\left(\bar{F}_{31}+\mathrm{k_s}\,\bar{E}_{15}\right)\eta\sum_{j=1}^{N}C_{ij}^{(1)}\varphi_j=-\bar{I}_3\,\omega^2\left[\sum_{j=1}^{N}\delta_{ij}\psi_j-\mu^2\sum_{j=1}^{N}C_{ij}^{(2)}\psi_j\right], \tag{35}$$

$$\bar{F}_{31}\,\eta\sum_{j=1}^{N}C_{ij}^{(1)}\psi_j+\bar{E}_{15}\left[\sum_{j=1}^{N}C_{ij}^{(2)}w_j+\eta\sum_{j=1}^{N}C_{ij}^{(1)}\psi_j\right]+\bar{X}_{11}\sum_{j=1}^{N}C_{ij}^{(2)}\varphi_j-\bar{X}_{33}\,\eta^2\sum_{j=1}^{N}\delta_{ij}\,\varphi_j=0 \qquad (36)$$

The boundary conditions (13-14) can also be approximated using three DQMs as:

(1) Clamped (C):

$$u_1=w_1=\psi_1=\varphi_1=0, \qquad at\,\zeta=0, \qquad (37)$$

$$u_N=w_N=\psi_N=\varphi_N=0, \qquad at\,\zeta=1, \qquad (38)$$

(2) Hinged (H):

$$\bar{F}_{31}\,\eta\sum_{j=1}^{N}\delta_{1j}\varphi_j-\mu^2\omega^2\left[\bar{I}_3\sum_{j=1}^{N}C_{1j}^{(2)}\,\psi_j+\bar{I}_1\,\eta\sum_{j=1}^{N}\delta_{1j}wj-\mu^2\bar{I}_1\eta\sum_{j=1}^{N}C_{1j}^{(2)}u_j\right]-\left(\bar{N}_T+\bar{N}_E\right)\eta\sum_{j=1}^{N}C_{1j}^{(2)}\,w_j=0, \qquad (39)$$

$$u_1=w_1=\varphi_1=0 \qquad \text{at}\ \zeta=0$$

$$\bar{F}_{31}\,\eta\sum_{j=1}^{N}\delta_{Nj}\varphi_j-\mu^2\omega^2\left[\bar{I}_3\sum_{j=1}^{N}C_{Nj}^{(2)}\psi_j+\bar{I}_1\eta\sum_{j=1}^{N}\delta_{Nj}wj-\mu^2\,\bar{I}_1\,\eta\sum_{j=1}^{N}C_{Nj}^{(2)}u_j\right]-\left(\bar{N}_T+\bar{N}_E\right)\eta\sum_{j=1}^{N}C_{Nj}^{(2)}w_j=0, \qquad (40)$$

$$u_N=w_N=\varphi_N=0 \qquad \text{at}\ \zeta=1$$

4.Numerical results

The present numerical results demonstrate convergence and efficiency of each one of the proposed schemes for vibration analysis of piezoelectric Nano elastic beam. For all results, the boundary conditions (37-40) are augmented in the governing equations (33-36). The computational characteristics of each scheme are adapted to reach accurate results with error of order $\leq 10^{-10}$. The obtained frequencies ω can be evaluated such as $\omega=\Omega\,L\sqrt{\frac{I_1}{A_{11}}}$.

For the present results, material parameters for the composite are listed in Table (1).

Table 1 Material property of piezoelectric Nano elastic beam [27-28].

Material properties	Elastic Constant (GPa)		Piezoelectric Constant (C/m^2)		Dielectric Constants (C/Vm) $*10^{-9}$		Thermal module (N/m^2K) $*10^5$	Density (Kg/m^3)
	C_{11}	C_{44}	e_{31}	e_{15}	ϵ_{11}	ϵ_{33}	λ_1	ρ
PTZ-4	132	26	-4.1	14.1	5.841	7.124	4.738	7500
BiTiO3-COFe2O4	226	44.2	-2.2	5.8	5.64	6.35	4.74	5550

For PDQM the problem is solved over a non-uniform grids, with Gauss – Chebyshev – Lobatto discretizations, such as [23]:

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right], \ (i = 1:N), \quad (41)$$

Where the dimensions of the grid (N*N) ranges from 3*3 to 15*15.

The obtained results agreed with previous analytical ones [29] over 7*7 grid size, as shown in Table 2.

Table 2 Comparison between the obtained normalized frequencies, due to PDQM, and the previous exact and numerical ones, for various grid sizes:clamped hinged nano elastic beam ($\Delta T = 0, V_0 = 0, L = 12nm, h = 2nm, \mu = 0, k_1 = k_2 = k_3 = 0$).

Normalized frequencies / Grid size N	ω_1	ω_2	ω_3	ω_4	ω_5
3	1.14592	2.8283	8.6761	-------	-------
5	0.6319	2.17824	3.1343	3.4421	6.9282
7	0.6323	1.7963	3.1416	3.4638	5.0183
9	0.6323	1.70999	2.94503	3.1416	4.2833
11	0.6323	1.740	3.1416	3.3040	3.8070
Exact results [29]	0.6323	-------	-------	-------	-------
PDQM [21] N=15	0.6323	-------	-------	-------	-------
Execution time (sec)	0.158375-- over 7 non-uniform grid				

o

For SincDQ scheme, the problem is solved over a regular grids ranging from 3*3 to 15*15. Table 3 shows convergence of the obtained results. They agreed with exact ones [29] over grid size ≥9*9. Also, this table shows that execution time of SincDQ scheme is less than that of PDQM. Therefor, it is more efficient than PDQM for vibration analysis of Nano elastic beam.

Table 3 Comparison between the obtained normalized frequencies, due to SINC DQM, and the previous exact and numerical ones, for various grid sizes:clamped hinged nano elastic beam ($\Delta T = 0, V_0 = 0, L = 12nm, h = 2nm, \mu = 0,\ k_1 = k_2 = k_3 0$).

Normalized frequencies / Grid size N	ω_1	ω_2	ω_3	ω_4	ω_5
3	1.2184	2.9554	8.9921	-------	--------
5	0.6255	1.5142	2.5907	3.0267	5.17044
7	0.6294	1.4906	2.5866	2.6281	3.803
9	0.6323	1.49676	2.5857	2.64027	3.8051
11	0.6323	1.49826	2.5856	2.64151	3.8095
Exact results [29]	0.6323	-------	-------	-------	-------
PDQM [21] N=15	0.6323	--------	-------	-------	-------
Execution time (sec)	0.142314-- over 9 uniform grid				

agreed with exact ones [29] over grid size ≥3*3 and bandwidth ≥ 3. Tables (4,5) show that execution time of DSCDQM-DLK is less than that of PDQM and SincDQM.

For DSCDQ scheme based on delta Lagrange kernel, the problem is also solved over a uniform grids ranging from 3*3 to 11*11. The bandwidth 2M+1 ranges from 3 to 11. Table 4 shows convergence of the obtained fundamental frequency which

Table 4.Comparison between the normalized fundamental frequency by using DSCDQM-DLK, band width (2M+1) and grid size N for clamped hinged nano elastic beam ($\Delta T = 0, V_0 = 0, L = 12nm, h = 2nm, \mu = 0,\ k_1 = k_2 = k_3 = 0$).

Fundamental frequency		DSCDQM-DLK				
Band width	N	3	5	7	9	11
2M+1 =3		0.6323	0.6323	0.6323	0.6323	0.6323
2M+1=5		0.6323	0.6323	0.6323	0.6323	0.6323
2M+1=7		0.6323	0.6323	0.6323	0.6323	0.6323
2M+1=9		0.6323	0.6323	0.6323	0.6323	0.6323
2M+1 =11		0.6323	0.6323	0.6323	0.6323	0.6323
Execution time (sec)		0.1400-- over 3 uniform grid				

Table 5 Comparison between the obtained normalized frequencies, due to DSCDQM-DLK , and the previous exact and numerical ones, for various grid sizes:clamped hinged nano elastic beam ($\Delta T = 0, V_0 = 0, L = 12nm,\ h = 2nm, \mu = 0, 2M + 1 = 3, k_1 = k_2 = k_3 = 0$).

Normalized frequencies / Grid size N	ω_1	ω_2	ω_3	ω_4	ω_5
3	0.6323	1.556178	3.121593	3.12533	4.38459
5	0.6323	1.556178	3.121593	3.12533	4.38459

7	0.6323	1.556178	3.121593	3.12533	4.38459
9	0.6323	1.556178	3.121593	3.12533	4.38459
Exact results [29]	0.6323	--------	--------	--------	-------
PDQM [21] N=15	0.6323	--------	--------	--------	-------
Execution time (sec)	0.1400-- over 3 uniform grid				

frequency to the exact and numerical ones [29,21] over grid size ≥3*3 , bandwidth ≥ 3 and regulization parameter σ = 2 h_x. Table 7 also ensures that execution time of this scheme is the least. Therefore, DSCDQM-RSK scheme is the best choice among the examined quadrature schemes for vibration analysis of Nano elastic beam.

For DSCDQ scheme based on regularized Shannon kernel (RSK), the problem is also solved over a uniform grids ranging from 3*3 to 15*15. The bandwidth 2M+1 ranges from 3 to 9 and the regularization parameter σ = r h_x ranges from 1h_x to 2 h_x , where h_x =1/N-1. Table 6 shows convergence of the obtained fundamental

Table 6.Comparison between the normalized fundamental frequency by using DSCDQM-RSK, band width (2M+1) regularization parameter σ and grid size N for clamped hinged nano elastic beam ($\Delta T = 0, V_0 = 0, L = 12nm,\ h = 2nm, \mu = 0, k_1 = k_2 = k_3 = 0$).

fundamental frequency	regularization parameter	DSCDQM-RSK				
N	2M+1	σ =1*h_x	σ =1.5*h_x	σ =1.8*h_x	σ =1.95*h_x	σ =2*h_x
3	3	1.22835	0.81413	0.67747	0.63488	0.6323
	5	1.22835	0.81413	0.67747	0.63488	0.6323
	7	1.22835	0.81413	0.67747	0.63488	0.6323
5	3	1.22835	0.81413	0.67747	0.63488	0.6323
	5	1.22835	0.81413	0.67747	0.63488	0.6323
	7	1.22835	0.81413	0.67747	0.63488	0.6323
7	3	1.22835	0.81413	0.67747	0.63488	0.6323
	5	1.22835	0.81413	0.67747	0.63488	0.6323
	7	1.22835	0.81413	0.67747	0.63488	0.6323
9	3	1.22835	0.81413	0.67747	0.63488	0.6323
	5	1.22835	0.81413	0.67747	0.63488	0.6323
	7	1.22835	0.81413	0.67747	0.63488	0.6323

Table 7 Comparison between the obtained normalized frequencies, due to DSCDQM-RSK and the previous exact and numerical ones, for various grid sizes:clamped hinged nano elastic beam ($\Delta T = 0, V_0 = 0, L = 12nm,\ h = 2nm, \mu = 0, 2M + 1 = 3, k_1 = k_2 = k_3 = 0$).

Normalized frequencies / Grid size N	ω_1	ω_2	ω_3	ω_4	ω_5
3	0.6323	1.556178	3.121593	3.12533	4.38459
5	0.6323	1.556178	3.121593	3.12533	4.38459

7	0.6323	1.556178	3.121593	3.12533	4.38459
9	0.6323	1.556178	3.121593	3.12533	4.38459
Exact results [29]	0.6323	--------	--------	-------	-------
PDQM [21] N=15	0.6323	--------	--------	-------	--------
Execution time (sec)	0.139032-- over 3 uniform grid				

Furthermore, a parametric study is introduced to investigate the influence of linear and nonlinear elastic foundations parameters, temperature change (ΔT °C), external electric voltage (V_0), nonlocal parameter (μ), length-to-thickness ratio (L/h), different boundary conditions and different materials on the values of natural frequencies and mode shapes. Tables (8-11) show that the fundamental frequency increases with increasing linear elastic foundation parameters. Also, the computations declare that the results do not affect significantly by nonlinear elastic foundation parameter k_3.

Figures (2-6) and tables (10-11) show that the fundamental frequency decrease with increasing temperature change (ΔT °C), external electric voltage (V_0), nonlocal parameter (μ) and length-to-thickness ratio (L/h) at different conditions of linear and nonlinear parameters of elastic foundation. As well as, Figs. (7-12) show the first three normalized mode shapes W and electrical potential (∅) with time at different materials, linear and nonlinear parameters of elastic foundation and boundary conditions. These figures show that the amplitudes of displacment W and electrical potential (∅) increase with increasing linear and nonlinear elastic foundation parameters. Furthermore, Figs. (4-7) show that the fundamental frequency and the normalized amplitude W for PTZ-4 material is higher than BiTiO3-COFe2O4 material.

Table. 8 Comparison between the normalized frequencies, elastic foundation parameters and the previous numerical ones for clamped clamped piezoelectric nano beam ($\Delta T = 0, V_0 = 0, L = 10nm$, $L/h = 5, k_3 = 0$).

Normalized frequencies			ω_1		ω_2		ω_3	
Elastic foundation parameters		Results	DSCDQM-RSK	PDQM [22]	DSCDQM-RSK	PDQM [22]	DSCDQM-RSK	PDQM [22]
k_2	k_1							
0	0		79.5849	79.5849	158.9145	158.915	226.9817	226.982
	5		79.6553	79.6553	158.9489	158.949	227.0057	227.006
	10		79.7256	79.7256	158.9833	158.983	227.0296	227.03

	15	79.7958	79.7958	159.0177	159.018	227.0535	227.054
	25	79.9361	79.9361	159.0864	159.086	227.1013	227.101
0.025	0	79.6294	79.6294	158.9935	158.994	227.0983	227.098
	5	79.7552	-----	159.1307	------	227.474	-----
	10	79.8126	------	159.1588	------	2.27493	------
	15	79.86996	------	159.1869	------	227.5126	------
	25	79.9845	------	159.243	------	227.5516	-----
0.05	0	79.6738	79.6738	159.0725	159.073	227.2149	227.215
	5	79.8676	-----	159.3305	-----	227.7683	-----
	10	79.92493	-----	159.3585	-----	227.7878	-----
	15	79.98219	-----	159.3866	-----	227.8072	-----
	25	80.0966	-----	159.4426	-----	227.8462	-----
0.1	0	79.7627	79.7627	159.2303	159.2303	227.4478	227.448
	5	80.03462	-----	159.789	-----	228.398	-----
	10	80.2141	-----	159.877	-----	228.45897	-----
	15	80.39327	-----	159.9645	-----	228.51999	-----
	25	80.572002	-----	160.1398	-----	228.64195	-----
0.15	0	79.8514	79.8514	159.3879	159.3879	227.6805	227.6805
	5	80.258170	-----	160.18625	-----	228.9842	-----
	10	80.4372	-----	160.2738	-----	229.045	-----
	15	80.61584	-----	1.60.3613	-----	229.1059	-----
	25	80.794074	-----	160.536	-----	229.22756	-----

Table.9 Comparison between the normalized nonlinear frequencies and elastic foundations for clamped clamped piezoelectric nano beam ($\Delta T = 0, V_0 = 0, L = 10nm, L/h = 5$).

Nonlinear Elastic parameters k_3		0.025		0.05		0.1		0.15	
Normalized Frequencies / Linear Elastic paramet		ω_1	ω_2	ω_1	ω_2	ω_1	ω_2	ω_1	ω_2
k_2	k_1								
0	0	79.5923	158.905	79.600	158.908	79.614	158.914	79.628	158.919
	5	79.6498	158.933	79.657	158.936	79.672	158.942	79.686	158.947
	10	79.7073	158.962	79.715	158.964	79.729	158.97	79.744	158.976
	15	79.7648	158.99	79.772	158.992	79.787	158.998	79.801	159.004
	25	79.8795	159.046	79.887	159.049	79.901	159.054	79.916	159.06
0.025	0	79.7050	159.105	79.712	159.108	79.727	159.114	79.741	159.119
	5	79.7625	159.133	79.77	159.136	79.784	159.142	79.799	159.148
	10	79.82	159.162	79.827	159.164	79.842	159.17	79.856	159.176
	15	79.877	159.19	79.884	159.192	79.899	159.198	79.913	159.204
	25	79.9918	159.246	79.999	159.248	80.014	159.254	80.028	159.26
0.05	0	79.8175	159.305	79.825	159.308	79.839	159.314	79.854	159.319
	5	79.8749	159.333	79.882	159.336	79.897	159.342	79.911	159.347
	10	79.9322	159.361	79.939	159.364	79.954	159.37	79.968	159.375
	15	79.9894	159.389	79.997	159.392	80.011	159.398	80.026	159.403
	25	80.1039	159.445	80.111	159.448	80.126	159.454	80.140	159.459

0.1	0	80.0419	159.704	80.049	159.707	80.064	159.712	80.078	159.718
	5	80.0991	159.732	80.106	159.735	80.121	159.740	80.135	159.746
	10	80.1562	159.76	80.164	159.763	80.178	159.768	80.193	159.774
	15	80.2133	159.788	80.221	159.791	80.235	159.796	80.25	159.802
	25	80.3275	159.844	80.335	159.847	80.349	159.852	80.364	159.858
0.15	0	80.2655	160.102	80.273	160.104	80.287	160.110	80.302	160.116
	5	80.3225	160.129	80.33	160.132	80.344	160.138	80.359	160.14
	10	80.3795	160.157	80.387	160.160	80.401	160.166	80.416	160.17
	15	80.4364	160.185	80.444	160.188	80.458	160.194	80.473	160.2
	25	80.5502	160.241	80.558	160.244	80.572	160.249	80.587	160.255

Table.10 Comparison between the normalized frequencies, boundary conditions and nonlocal parameter (μ) for piezoelectric nano beam $(\Delta T = 0, V_0 = 0, \frac{L}{h} = 20, k_1 = 10, k_2 = 0.025, k_3 = 0.05)$.

Normalized frequencies		ω_1	ω_2	ω_3	ω_4	ω_5
B.C	μ					
CH	0	0.3771	0.8337	1.5362	2.4332	3.1416
	0.05	0.3766	0.8089	1.4166	2.1054	2.8088
	0.1	0.3754	0.7537	1.2067	1.6503	2.0571
	0.15	0.3736	0.6976	1.0446	1.3615	1.6412
	0.2	0.3715	0.6534	0.9398	1.1893	1.3989
CC	0	0.4430	0.9629	1.7064	2.6248	3.1416
	0.05	0.4432	0.9326	1.5696	2.2658	2.9612
	0.1	0.4438	0.8658	1.3326	1.7719	2.1603
	0.15	0.4446	0.7988	1.1532	1.45914	1.7118
	0.2	0.4453	0.7465	1.03995	1.2696	1.4506
HH	0	0.3293	0.717	1.3718	2.242	3.1416
	0.05	0.3285	0.697	1.2688	1.9449	2.6508
	0.1	0.3264	0.652	1.0851	1.5281	1.9399
	0.15	0.3233	0.6051	0.9402	1.261	1.5452
	0.2	0.31998	0.567	0.8447	1.1021	1.3194

Table. 11 Comparison between the normalized frequencies, boundary conditions and length-to-thickness ratio (L/h) for piezoelectric nano beam $(\Delta T = 0, V_0 = 0, h = 2nm, \mu = 0.1, k_1 = 10, k_2 = 0.025, k_3 = 0.05)$.

Normalized frequencies		ω_1	ω_2	ω_3	ω_4	ω_5
B.C	L/h					
CC	6	0.7973	1.6219	2.4572	2.758	2.9033
	8	0.6685	1.4081	2.1237	2.4449	2.6259
	12	0.513	1.1052	1.7075	2.1882	2.5314
	16	0.4512	0.939	1.4615	1.9295	2.3183
	20	0.4438	0.8658	1.3326	1.7719	2.1603
	30	0.5345	0.9118	1.3326	1.7463	2.1317
CH	6	0.6078	1.4739	2.4192	1.8264	3.0025
	8	0.4986	1.2476	1.9866	2.4179	2.6372
	12	0.3866	0.9538	1.5646	2.0886	2.4762

	16	0.3593	0.8083	1.3254	1.8093	2.2275
	20	0.3754	0.7537	1.20675	1.6503	2.05711
	30	0.4894	0.8227	1.2206	1.6258	2.0217
HH	6	0.4327	1.3085	2.1982	2.4778	2.5803
	8	0.34998	1.0766	1.8455	2.3652	2.6481
	12	0.2861	0.8039	1.41798	1.9725	2.4066
	16	0.2914	0.6852	1.1902	1.6837	2.11944
	20	0.3264	0.652	1.08514	1.5281	1.9399
	30	0.4561	0.7457	1.117	1.5105	1.8952

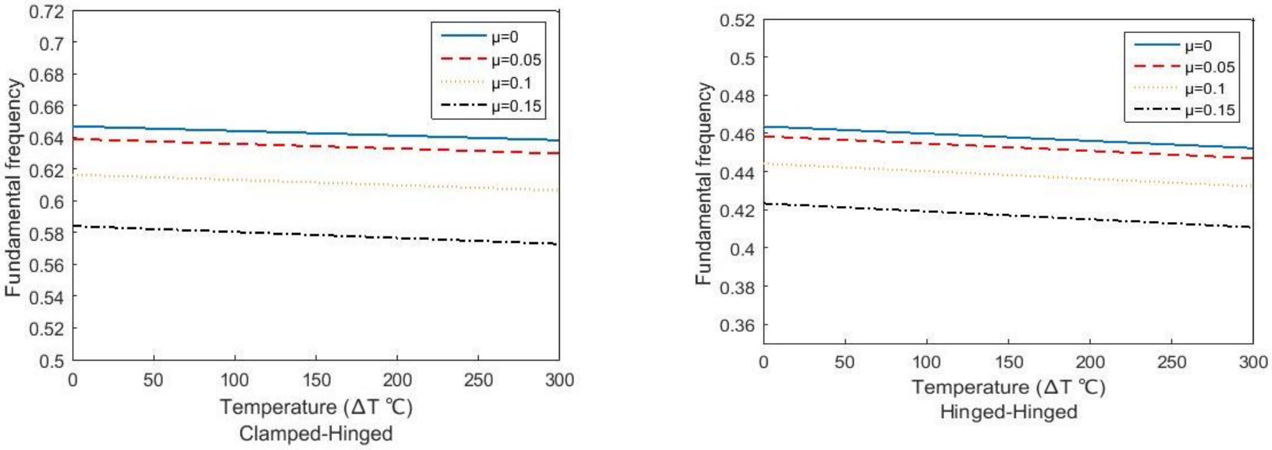

Fig.2.Variation of fundamental frequency with temperature (ΔT °C), nonlocal parameter (μ) and different boundary conditions for Nano elastic beam. ($V_0 = 0,\ L/h = 6,\ k_1 = 25, k_2 = 0.05, k_3 = 0.025$).

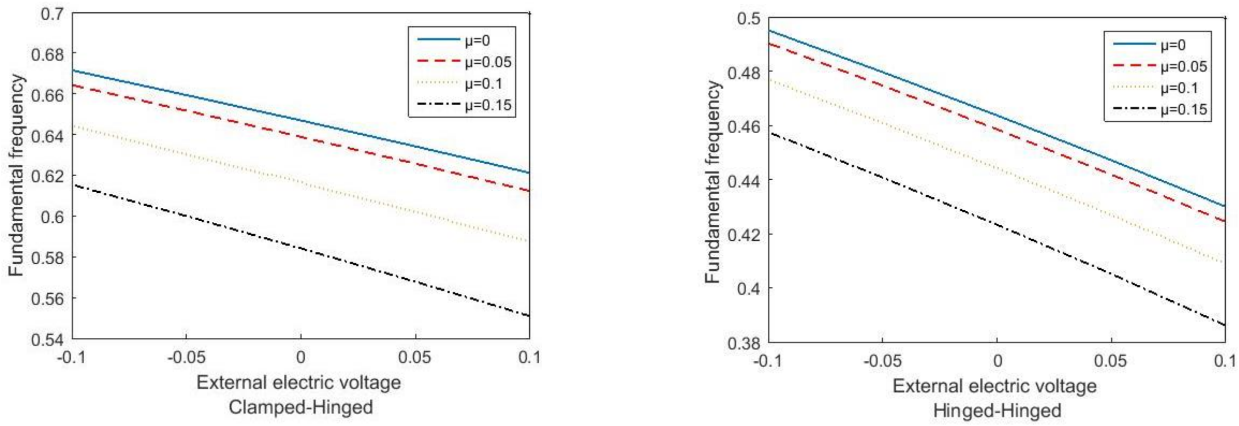

Fig.3.Variation of fundamental frequency with external electric voltage (V_0), nonlocal parameter (μ) and different boundary conditions for Nano elastic beam. $\left(\Delta T = 0,\ \frac{L}{h} = 6,\ k_1 = 25, k_2 = 0.05, k_3 = 0.025 \right)$.

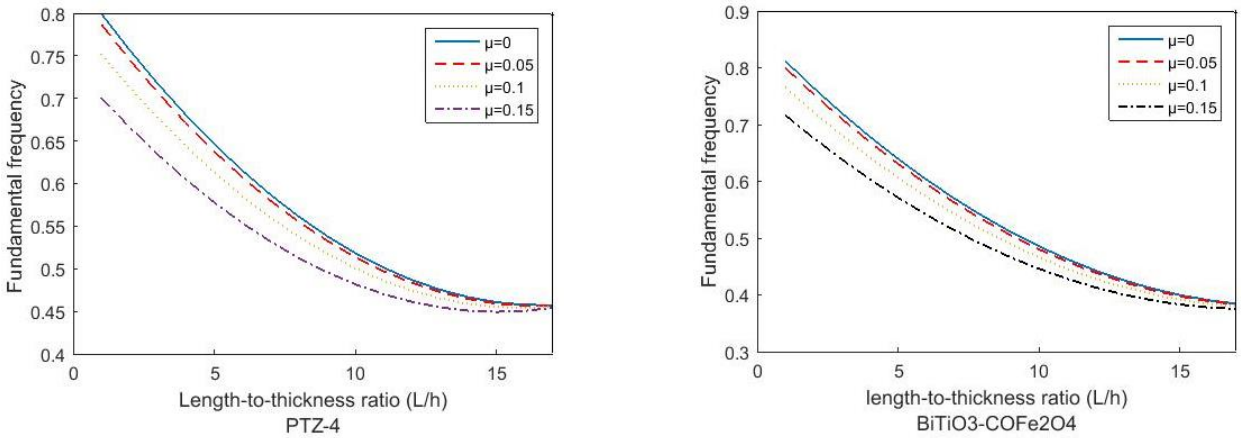

Fig.4.Variation of fundamental frequency with length-to-thickness ratio (L/h), nonlocal parameter (μ) and different materials for hinged Nano elastic beam. ($V_0 = 0,\ \Delta T = 0,\ k_1 = 25, k_2 = 0.05, k_3 = 0.025$).

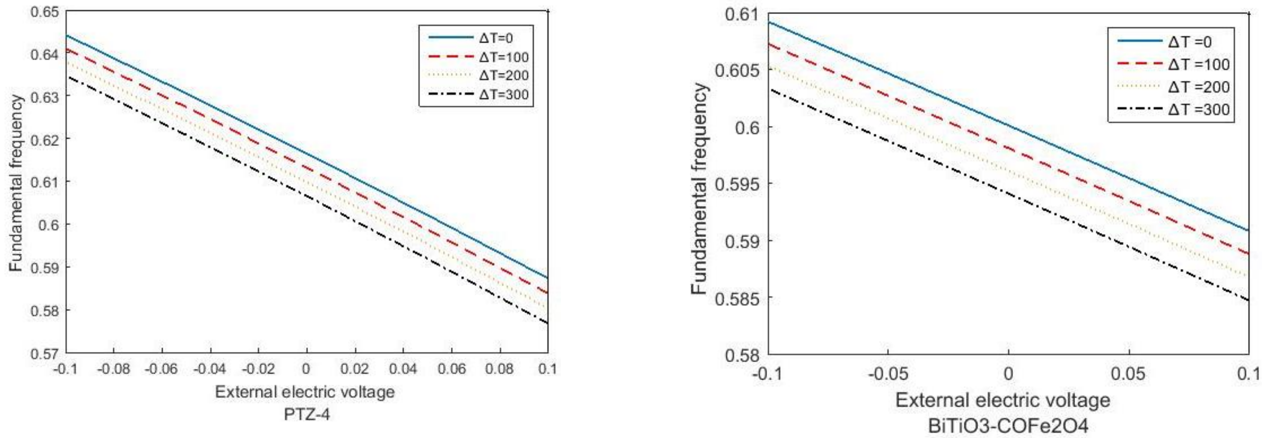

Fig.5.Variation of fundamental frequency with external electric voltage (V_0), temperature (ΔT °C) and different materials for hinged Nano elastic beam. ($L/h = 6\ ,\ \ \mu = 0.1\ ,\ k_1 = 25, k_2 = 0.05, k_3 = 0.025$).

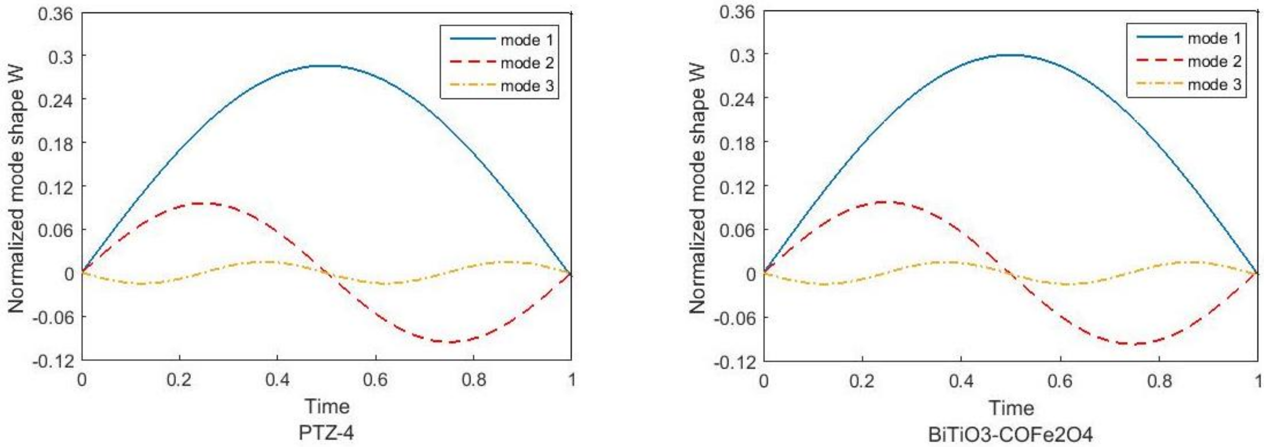

Fig.7. Variation of normalized mode shape W with time for first three modes at different material for clamped hinged Nano elastic beam. ($V_0 = 0, \Delta T = 100, L/h = 6$, $\mu = 0.1$, $k_1 = k_2 = k_3 = 0$).

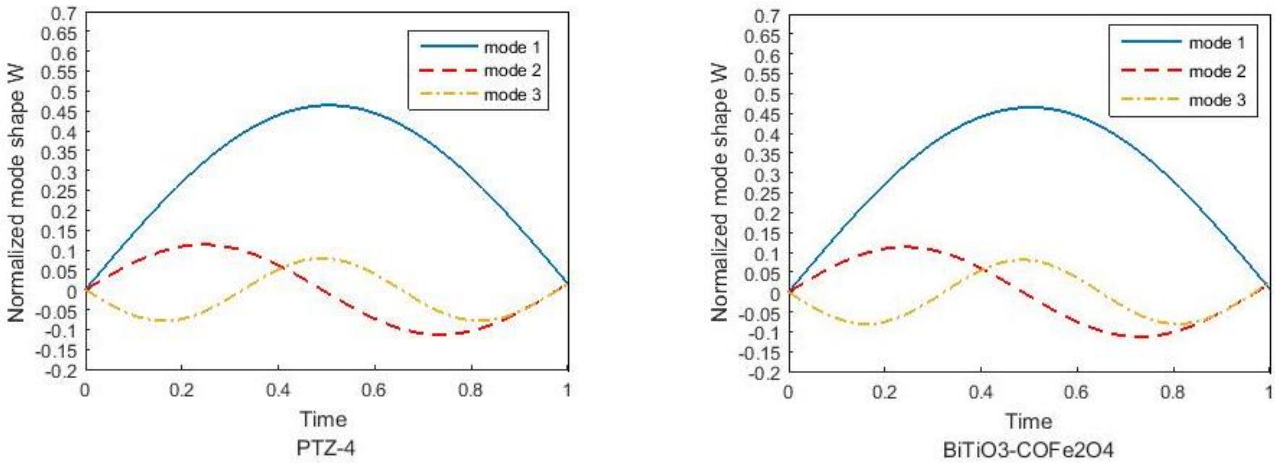

Fig.9. Variation of normalized mode shape W with time for first three modes at different material for clamped hinged Nano elastic beam. ($V_0 = 0, \Delta T = 100, L/h = 6$, $\mu = 0.1$, $k_1 = 25$, $k_2 = 0.15$, $k_3 = 0.5$).

5.Conclusion

Three Different Quadrature schemes have been successfully applied for free vibration analysis of piezoelectric Nano elastic beam. A matlab program is designed for each scheme such that the maximum error (comparing with the previous exact results) is $\leq 10^{-10}$. Also, Execution time for each scheme, is determined. It is concluded that discrete singular convolution differential quadrature method based on regularized Shannon kernel (DSCDQM-RSK) with grid size ≥ 3, bandwidth 2M+1 ≥ 3 and regulization parameter $\sigma = 2*h_x$ leads to best accurate efficient results for the concerned problem. Iterative quadrature technique is used to obtain nonlinear algebraic system. Based on this scheme, a parametric study is introduced to investigate the influence of linear and nonlinear elastic foundation, geometric characteristics and type of material of the vibrated beam, on results. It is aimed that these results may be useful for design purpose, electromechanical applications and many fields of industrial revolution.

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