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DYNAMIC ANALYSIS OF CABLE-STAYED BRIDGES UNDER VARIOUS TRAFFIC LOAD SPEEDS

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Abstract

This paper presents the study of traffic load variations effect on the dynamic analysis of cable-stayed bridges. Time histories of displacement, velocity, acceleration, normal force, and bending moment are presented for different traffic load speeds. This study is concerned with harp bridges having five spans considering single plane of cables with 140 m for exterior spans, and 280 m for interior spans. The total length of the bridge is 1120 m. The own weight of all structural elements, and traffic load including impact are taken into account. In the dynamic analysis, the energy method, based on the minimization of the total potential energy (TPE) of structural elements, via conjugate gradient technique is used. The procedure is carried out using the iterative steps to acquire the final configuration. All prepared computer programs in FORTRAN language for this work and their verification is written by [1]. The conclusion, which have been drawn from the present work are outlined.

Keywords: Dynamic analysis, Traffic load speeds, Conjugate gradient method, Cable-Stayed Bridges.

1- Introduction

Cable stayed bridges are the bridges that have one or more towers from which cables support the floor beams. There are three major classes of cable stayed bridges harp, radiating and fan. In the harp bridges, the cables are nearly parallel. The Radiating is like the harp but the spacing between cables on the deck not equal the spacing on the tower. In the fan bridges, the cables are connected to the top of the towers. In the medium lengths, the harp bridge is preferred. The cable-stayed bridges are optimum for spans longer than cantilever bridges and shorter than suspension bridges.

Traffic load speed variations have a great effect on the dynamic analysis of cable-stayed bridges.

Many studies on this type of bridge have been carried out in the last half-century. ***Chatterjee, Datta and Surana***, presented a continuum approach for determining the vibration of cable-stayed bridges under the passage of moving vehicle and the responses were analyzed for a parametric study. The analysis was performed in time domain and it considered the effect of flexural of pylons, interaction between vehicle and bridge and coupling of vertical and torsional motions of the floor beam due to eccentrically placed vehicles. Results of the study helped to understand the influence of some important bridge and vehicle parameters on dynamic behavior of cable-stayed bridges [2]. ***Fuheng Yang and Ghislain A.Fonder*** discussed a modeling method for cable-stayed bridges with non-linear cables that is used for the dynamic response under moving loads. Individual stay cables were modeled by short catenary cable elements in which the non-linearity due to large displacements but small deformations and the non-linearity due to the vibrations if the tension were taken into consideration [3]. ***W.H.Guo and Y.L.Xu***, presented a fully computerized approach for assembling equations of motion of any type of coupled vehicle bridge system. A case study of a real long span cable-stayed bridge with a group of moving heavy vehicles demonstrated that the fully computerized approach and the associated computer programs provide an efficient and convenient tool for studying the interaction problems of large complicated bridges with various types of running vehicles [4]. ***E.Savin*** derived analytical expressions of the dynamic amplification factor and the characteristic response spectrum for weakly damped beams with various boundary conditions subjected to point loads moving at constant speed [5]. He ***Zhang and Xu Xie***, characterized stays cables, the primary load carrying component of cable-stayed bridges, by high flexure that increases with the span of the bridge. This made stay cables vulnerable to local vibrations which may have significant effects on the dynamic responses of long cable-stayed bridges so, they studied the dynamic responses of cable-stayed bridges under vehicular loads using the finite element method, while the local vibration of stay cables was analyzed using the substructure method. The study showed that stay cables undergo large displacements in the primary mode of the whole bridge although, in general, a cable's local vibrations were not obvious [6]. ***Fabrizio Greco, Paolo Lonetti and Arturo Oascuzzo***, investigated the dynamic behavior of cable-stayed bridges subjected to moving loads and affected by an accidental failure in the cable system. The aim of the study was to quantify, numerically, the dynamic amplification factors of typical kinematic and stress design variables, by means of a parametric study developed in terms of the structural characteristics of the bridge components [7].

In the present work, Energy method is used for the analysis, and it is a unifying approach to the analysis of both linear and non-linear structures by considering the determination of equilibrium as an iterative process of minimizing the total potential energy, the position of equilibrium being reached when the total potential energy is minimum [8], [9], [10], [11], and [12]. A summary with a step-by-step iterative procedure is presented.

Step-by-step static response analysis by minimization of the total potential energy

The point at which W (total potential work) is a minimum defines the equilibrium position of the loaded structure. Mathematically, the equilibrium condition in the i-direction at joint j may be expressed as: x_{ji}

$$\frac{\partial W}{\partial x_{ji}} = [g_{ji}] = 0 \quad , i = 1,2 \; and \; 3 \qquad (1)$$

Where:

x_{ji} = The displacement of joint j corresponding to a particular degree of freedom, direction i.

g_{ji} =The corresponding gradient of the energy surface.

The location of the position where W is the minimum is achieved by moving down the energy surface along descent vector v a distance S_v until W is a minimum in that direction, that is, to a point where:

$$\frac{\partial W}{\partial s} = 0 \qquad (2)$$

From this point a new descent vector is calculated and the above process is repeated. The method is mathematically expressing this displacement vector at the (k+1) th iteration as:

$$[X]_{k+1} = [X]_k + S_k v_k \qquad (3)$$

Where:

v_k = The descent vector at the kith iteration from x_k in displacement space

S_k = the step length determining the distance along v_k to the point where W is a minimum.

Summary of the iterative procedures

The main steps in the iterative processes required to achieve structural equilibrium by minimization of total potential energy may be summarized as follows:

First, before the start of the iteration scheme

a) Calculate the tension coefficients for the pretension forces in the cable by:

$$t_{jn} = \left[\left(T_o + \frac{EA}{L_o}\right)/L_o\right]_{jn} \qquad (4)$$

Where:

e = the elongation of cable elements due to applied load only;

t_{jn} = the tension coefficient of the force in member jn ;

T_o = initial force in a pin-jointed member or cable link due to pretension;

E = modulus of elasticity;

A = area of the cable element; and

L_o = the unstrained initial length of the cable link

b) The elements in the initial displacement vector [X_o] are considered as zero.

c) Calculate the lengths of all the elements in the pretension structure using:

$$L_o^2 = \sum_{i=1}^{3}\left(X_{ni} - X_{ji}\right)^2 \qquad (5)$$

Where:

X = element in displacement vector due to applied load only.

d) To meet the convergence with minimum time, the technique of scaling matrix is used [13], [14] and [15]. The elements in the scaling matrix are given by:

$$H = diag\{k_{11}^{-1/2}, k_{22}^{-1/2} \ldots\ldots, k_{nn}^{-1/2}\} \qquad (6)$$

Where:

n = total number of degrees of freedom of all joint;

k = the 12 x 12 matrix of the element in global co-ordinates.

The steps in the iterative procedure then are summarized as

Step (1) Calculate the elements in the gradient vector of the TPE, using:

$$g_{ni} = \sum_{n=1}^{f_n}\sum_{r=1}^{12}(k_{nr}x_r)_n - \sum_{n=1}^{P_n}\left(t_{jn}\left(X_{ni} + x_{ni} - X_{ji} - x_{ji}\right)\right) - F_{ni} \qquad (7)$$

Step (2) Calculate the Euclidean norm of the gradient vector, $R_k = [g_k^T g_k]^{1/2}$, and check if the problem has converged. If $R_k \leq R_{min}$ stop the calculations and print the results. If not proceed to step (3).

Step (3) Calculate the elements in the descent vector, v using:

$$[v]_{k+1} = -[H][g]_{k+1} + \beta_k [v]_k \quad (8)$$

Where: $[v]_o = -[g]_o$ (9)

And $\beta_k = \frac{[g]_{k+1}^T [H]^T [H][g]_{k+1}}{[g]_k^T [H]^T [H][g]_k}$

$$= \frac{[g]_{k+1}^T [\widehat{K}][g]_{k+1}}{[g]_k^T [\widehat{K}][g]_k} \quad (10)$$

Step **(4)** Calculate the coefficients in the step-length polynomial form:

$$C_4 = \sum_{n=1}^{P} (EAa_3^2/2L_o^3)_n \quad (11a)$$

$$C_3 = \sum_{n=1}^{P} (EAa_2 a_3/L_o^3)_n \quad (11b)$$

$$C_2 = \sum_{n=1}^{P} \left[t_0 a_3 + \frac{EA(a_2^2 + 2a_1 a_3)}{2L_o^3}\right]_n + \sum_{n=1}^{f} \sum_{s=1}^{12} \sum_{r=1}^{12} \left(\frac{1}{2} v_s k_{sr} v_r\right)_n \quad (11c)$$

$$C_1 = \sum_{n=1}^{P} \left[t_0 a_2 + \frac{EAa_1 a_2}{L_o^3}\right]_n + \sum_{n=1}^{f} \sum_{s=1}^{12} \sum_{r=1}^{12} (x_s k_{sr} v_s)_n - \sum_{n=1}^{N} F_n v_n \quad (11d)$$

Where:

$$a_1 = \sum_{i=1}^{3} \left[\left(X_{ni} - X_{ji}\right) + \frac{1}{2}\left(x_{ni} - x_{ji}\right)\right]\left(x_{ni} - x_{ji}\right) + L_o^2 \frac{T_o}{EA} \quad (12a)$$

$$a_2 = \sum_{i=1}^{3} \left[\left(X_{ni} - X_{ji}\right) + \left(x_{ni} - x_{ji}\right)\right]\left(v_{ni} - v_{ji}\right) \quad (12b)$$

$$a_3 = \sum_{i=1}^{3} \frac{1}{2}\left(v_{ni} - v_{ji}\right)^2 \quad (12c)$$

Where:

f = Number of flexural members;

P = Number of pin-jointed members and cable links;

F = Element in applied load vector; and

K_{sr} = Element of stiffness matrix in global co-ordinates of a flexural element.

Step (5) Calculate the step-length S using Newton's approximation formula as:

$$S_{k+1} = S_k - \frac{4C_4 S^3 + 3C_3 S^2 + 2C_2 S + C_1}{12C_4 S^2 + 6C_3 S + 2C_2} \quad (13)$$

Where:

k is an iteration suffix and $S_{k=0}$ is taken as zero

Step (6) Update the tension coefficients using the following equation:

$$(t_{ab})_{k+1} = (t_{ab})_k + \frac{EA}{(L_o^3)_{ab}}(a_1 + a_2 s + a_3 s^2)_{ab} \quad (14)$$

Step (7) Update the displacement vector using equation (4).

Step (8) Repeat the above iteration by returning to step (1).

2- Bridge Description

With reference to Fig. (1), which shows the configuration of a five-span cable-stayed bridge. The bridge has two equal exterior spans of 140 m, each, and the interior spans are 280 m long, each. The deck girder has a total span of 1120 m. The bridge is symmetrical and composed of three major elements: (a) the deck girder, (b) four pylons and (c) eleven cables on each side of the pylons. The cables were 6x37 classes IWRC [16] of zinc-coated bridge ropes. All cables have an area of 61.94 cm^2, diameter of 10.16 cm, own weight of 48.96 kg/m, modulus of elasticity of 1584 t/cm^2, and the maximum failure load of 925 tons. The initial tension in all cables was taken as 30 % of the maximum failure load (925 tons) for the 1st iteration then circle of solution technique is used with 20 cycles [1]. All section properties for cables, pylons and floor beam are given in Table (1).

3- Analysis Considerations

The dynamic analysis of the harp model with the given dimension, degree of freedom and properties is carried out by the energy method. The total equivalent traffic load including impact effect on the bridge is taken 5.28 t/m'. The analysis was carried out for 90 seconds, and a time step of 0.02 second with 4500 time steps, also the natural frequency used was taken 0.57994 cycle/second, and the damping ratio was taken 0.015. Excel sheets were prepared for the loads on the floor beam for the analysis. They contain load value on the corresponding degree of freedom of the floor beam at each time step. Those sheets were prepared for the various speeds taken into consideration (60, 90, 120, and 300 km/hr.). All results are showed in figures.

4- Analysis of Results

Figures (2) to (25) showed some of the results obtained from the analysis:

1. Figs. (2) to (4) showed that, the displacements are smaller at low speeds but with long period, while at high speeds, displacements become larger with small period. At the second span's midpoint, maximum displacement occurs after 23.72, 16.92, 12.2 and 5.4 seconds from the beginning of crossing the train through the bridge under the speed of 60, 90, 120 and 30 km/hr. respectively, when at the second one's, it occurs after 49.2, 27.12, 20.2 and 8.2 seconds, and at the third one's, it occurs after 56.6, 40.2, 28 and 12 seconds.
2. Figs. (5) to (7) showed that, at the first span's midpoint, maximum velocity occurs after 15.74, 12.2, 8.5 and 3.4 seconds, when at the second one's, it occurs after 35.2, 22.1, 16.4 and 6.8 seconds, and at the third one's, it occurs after 47.5, 32.6, 25.1 and 9.8 seconds from the beginning of crossing the train through the bridge under the speed

of 60, 90, 120 and 300 km/hr. respectively.

3. Figs. (8) to (19) showed that, at the first span's midpoint, acceleration variation has a range from -1.1 to 1, -7 to 7, -7.2 to 8 and -1.4 to 1.4 m/s^2, when at the second one's, from -0.92 to 0.8, -7 to 7.6, -1.4 to 1.4 and -7.9 to 7.9 m/s^2, and at the third one's, from -6.2 to 7, -6.2 to 7, -1.2 to 1.35 and -6.8 to 7 m/s^2 under the speed of 60, 90, 120 and 300 km/hr. respectively.
4. Figs. (20) to (22) showed that, maximum normal force is -486.82, -480.57, -457.92 and -698.54 tons, when at the second one's, it is -604.5, -559.64, -644.167 and -702.3 tons, and at the third one's, it is -367.94, -427.78, -438.928 and -841.6 tons under the speed of 60, 90, 120 and 300 km/hr. respectively.
5. Figs. (23) to (25) showed that, at the first span's midpoint, maximum bending moment is -5662.46, -5624.62, -5725.58 and -6515.35 t.m, when at the second one's, it is -4757.88, -4720.11, -4832.91 and -5474.4 t.m, and at the third one's, it is -4000, -4000, -3831 and -4850 t.m under the speed of 60, 90, 120 and 300 km/hr. respectively.

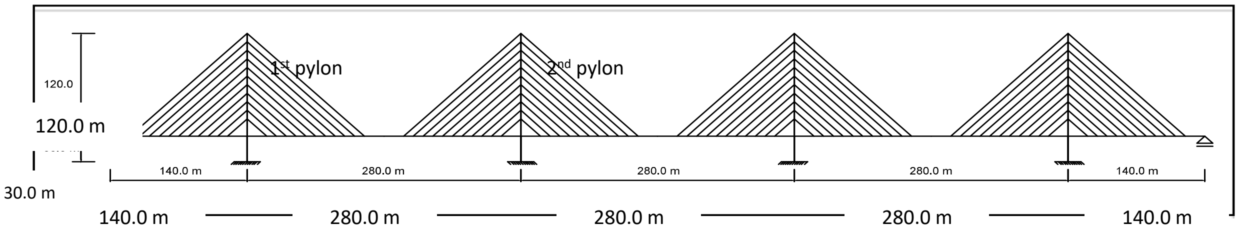


Fig. (1): Configuration of the Bridge

Table (1): Properties of Sections used

Strctural Element	Description of Structrual Elements	Properties of Sections					Loads	
		Moldulus of Elasticity	Area	Inertia @ X	Inertia @ Y	Inertia @ Z	Dead Load	Live Load
		t/cm2	m2	m4	m4	m4	t/m	t/m
Pylon	Hollow rectangular R.C. section	300	5.76	17.66	7.4	15.9	14.4	0
Deck	Steel box girder	2100	0.625	1.14	30.5	31.64	5.78	5.28
Cables	Spiral strand	1584	0.00619354				0.04896167	0

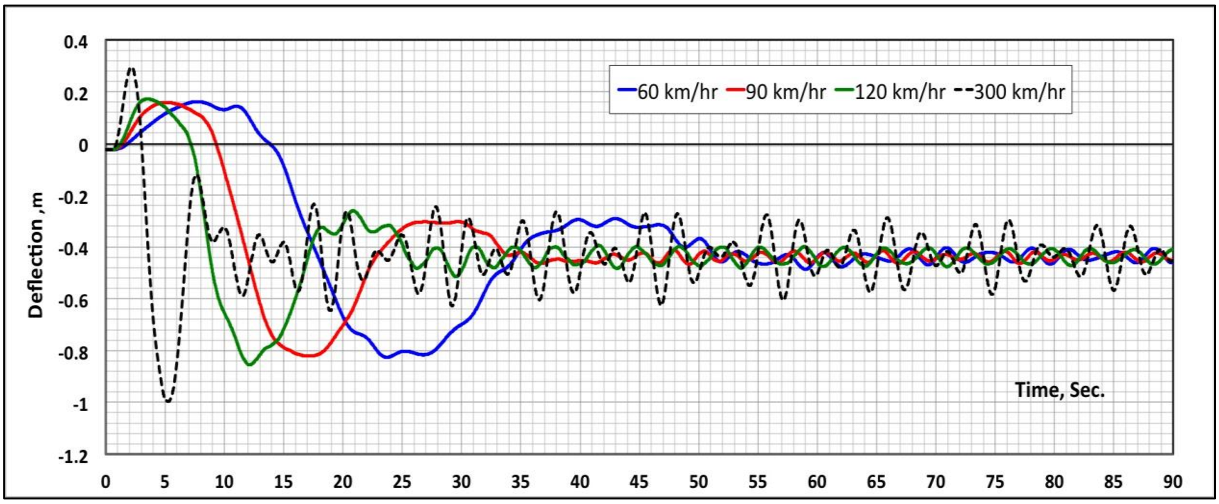


Fig. (2): Variation of displacement at mid second span for various considered speeds

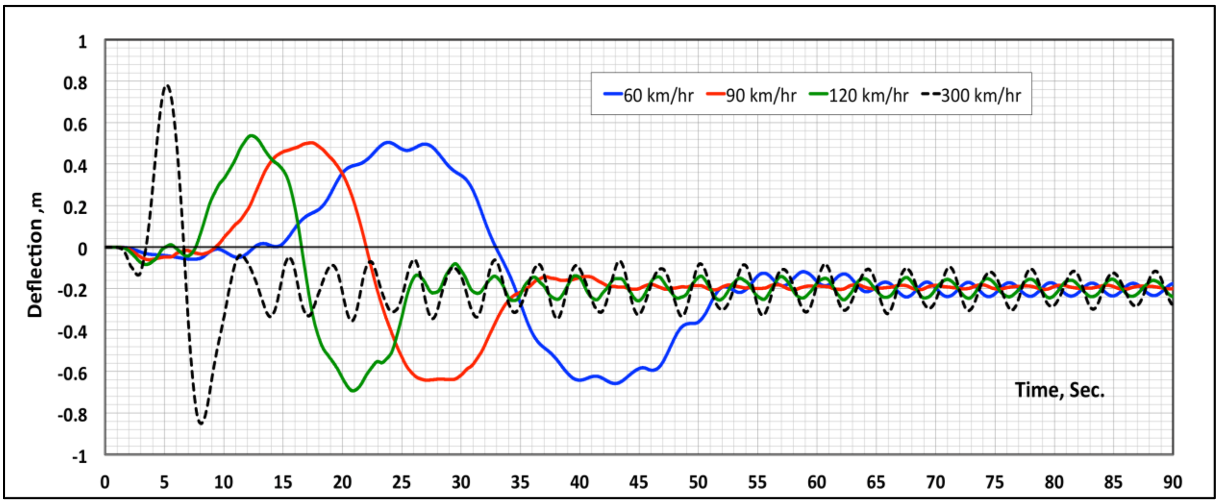


Fig. (3): Variation of displacement at mid third span for various considered speeds

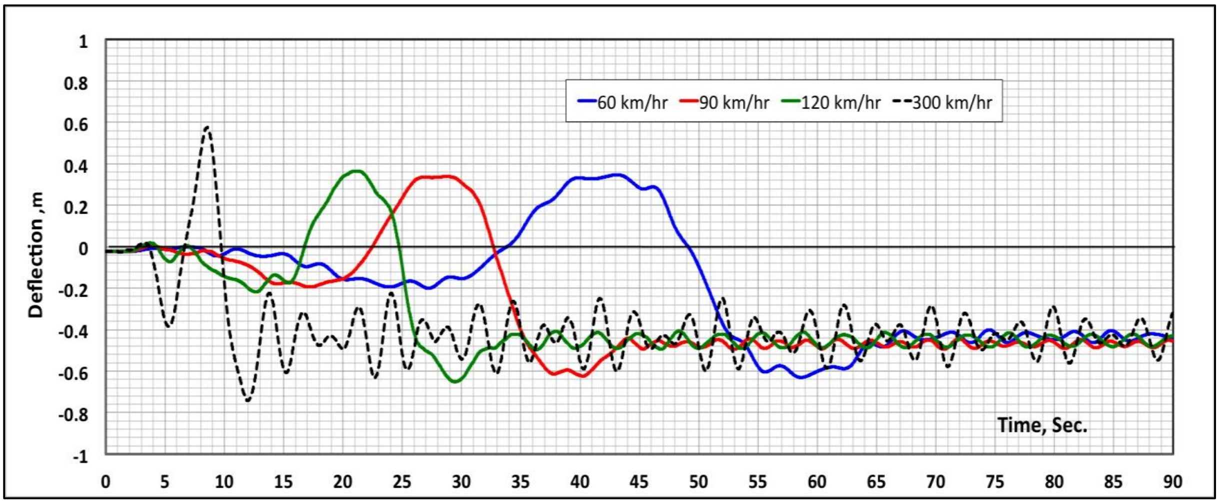


Fig. (4): Variation of displacement at mid fourth span for various considered speeds

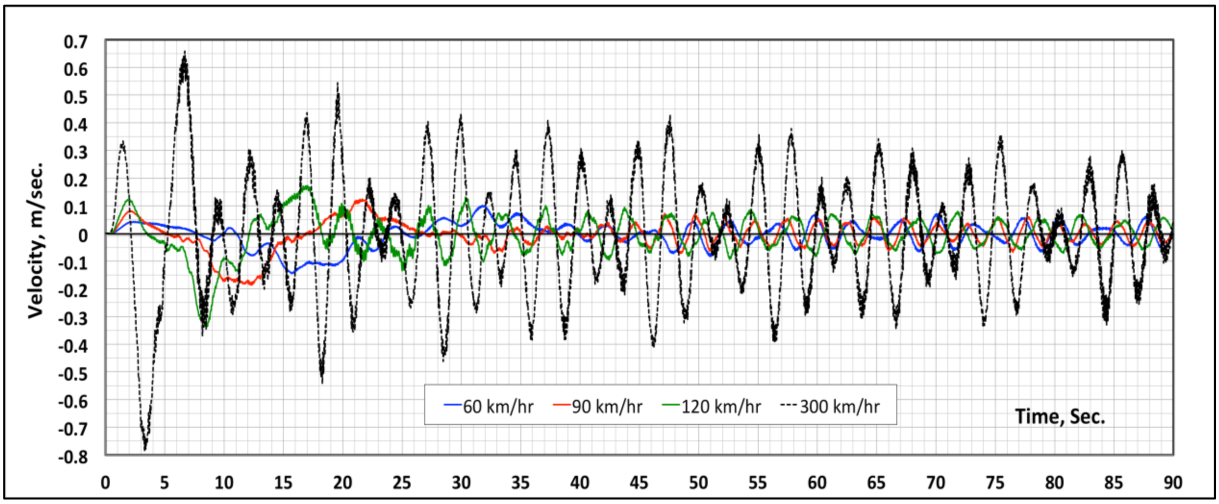


Fig. (5): Variation of velocity at mid second span for various considered speeds

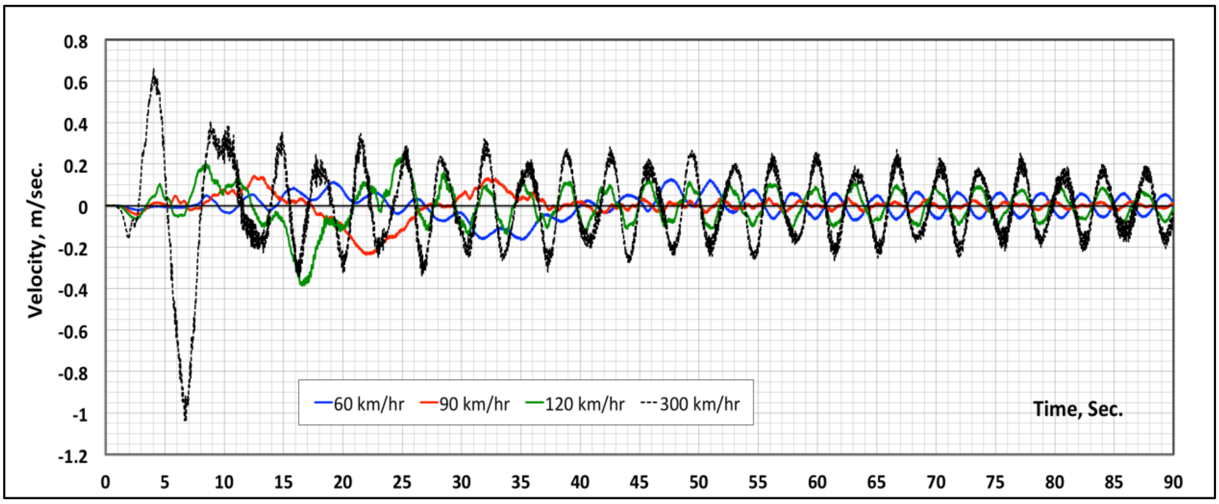


Fig. (6): Variation of velocity at mid third span for various considered speeds

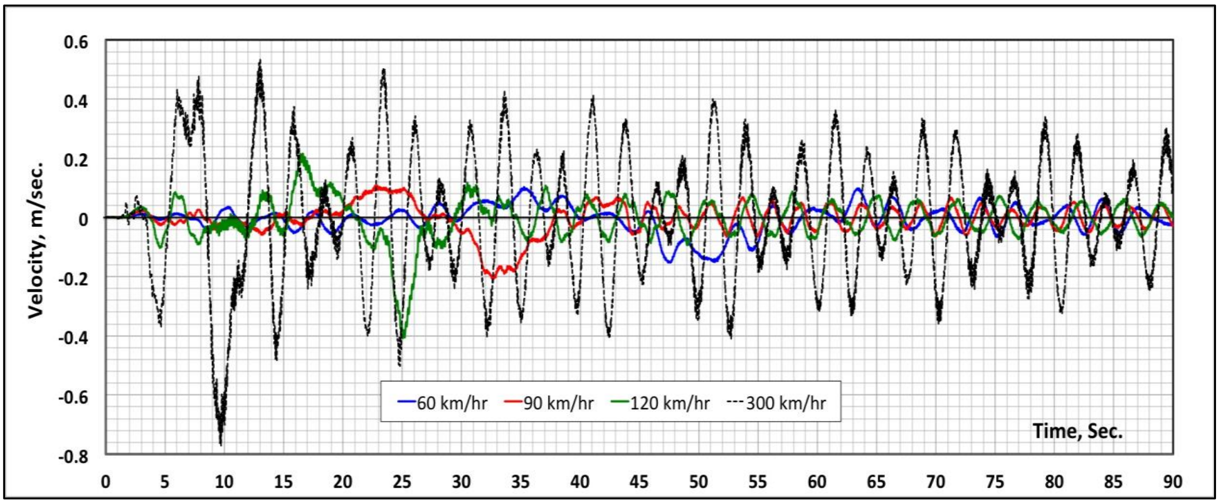


Fig. (7): Variation of velocity at mid fourth span for various considered speeds

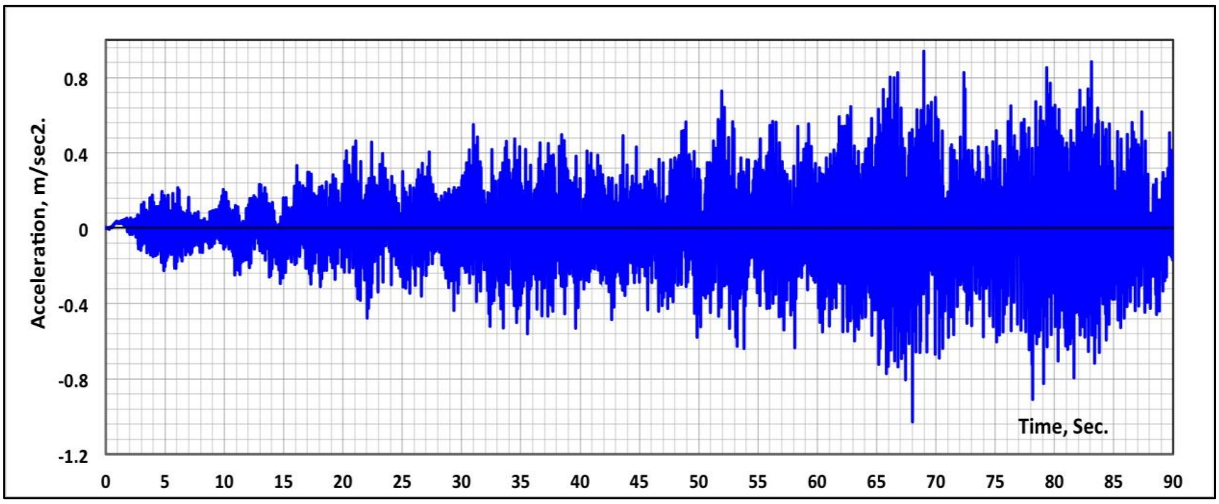


Fig. (9): Acceleration variation on the third span's mid-joint under the speed of 60km/hr.

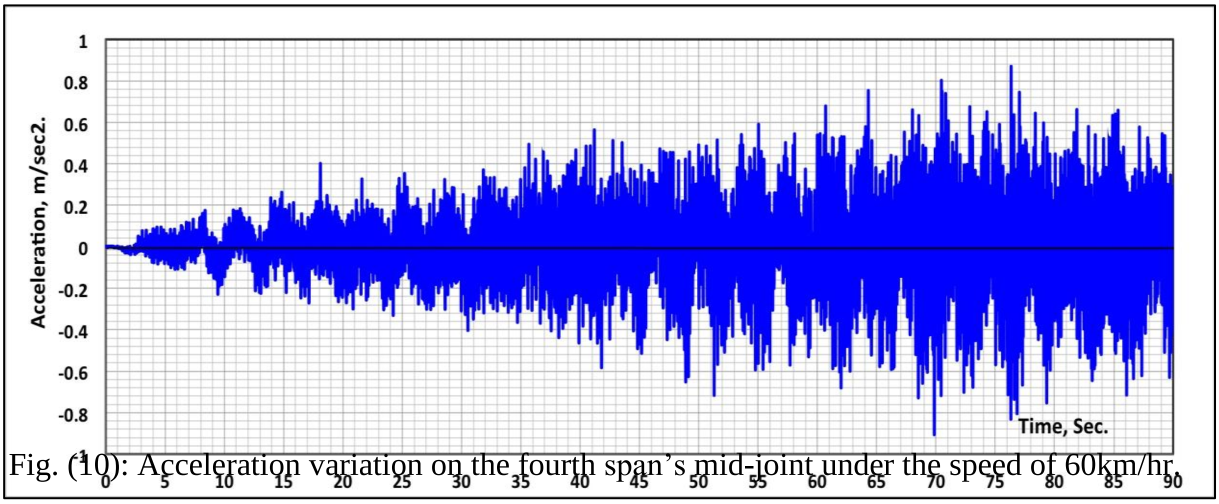
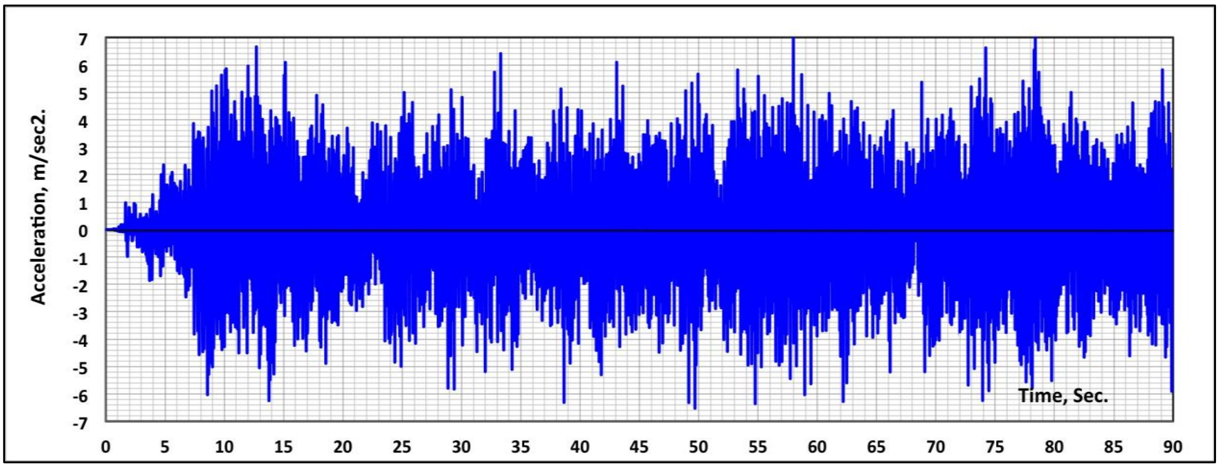


Fig. (10): Acceleration variation on the fourth span's mid-joint under the speed of 60km/hr.

Fig. (8): Acceleration variation on the second span's mid-joint under the speed of 60km/hr.



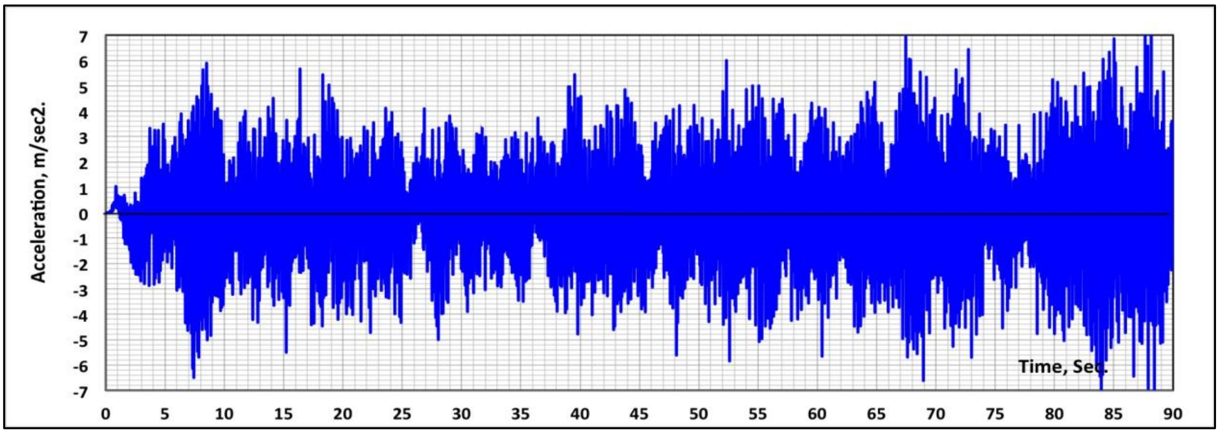


Fig. (12): Acceleration variation on the third span's mid-joint under the speed of 90km/hr.

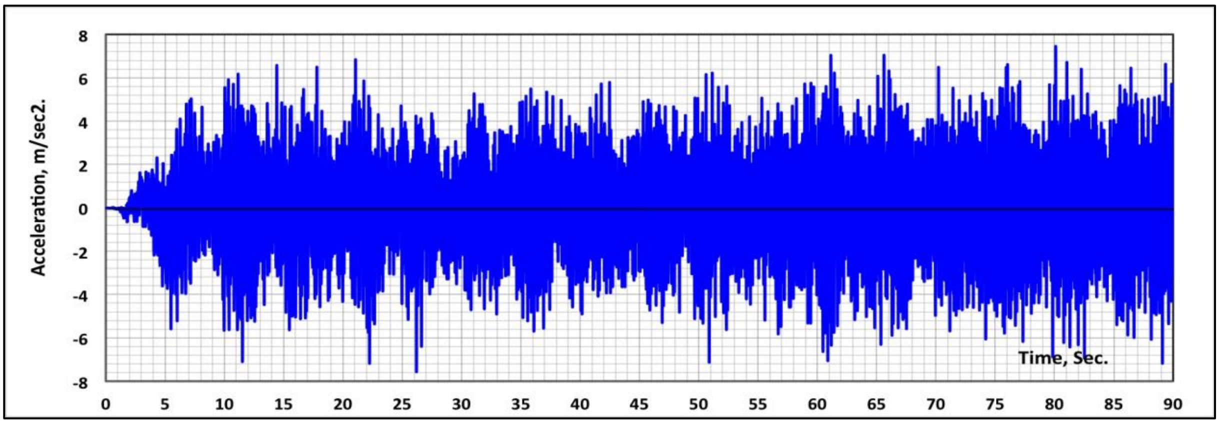


Fig. (13): Acceleration variation on the fourth span's mid-joint under the speed of 90km/hr.

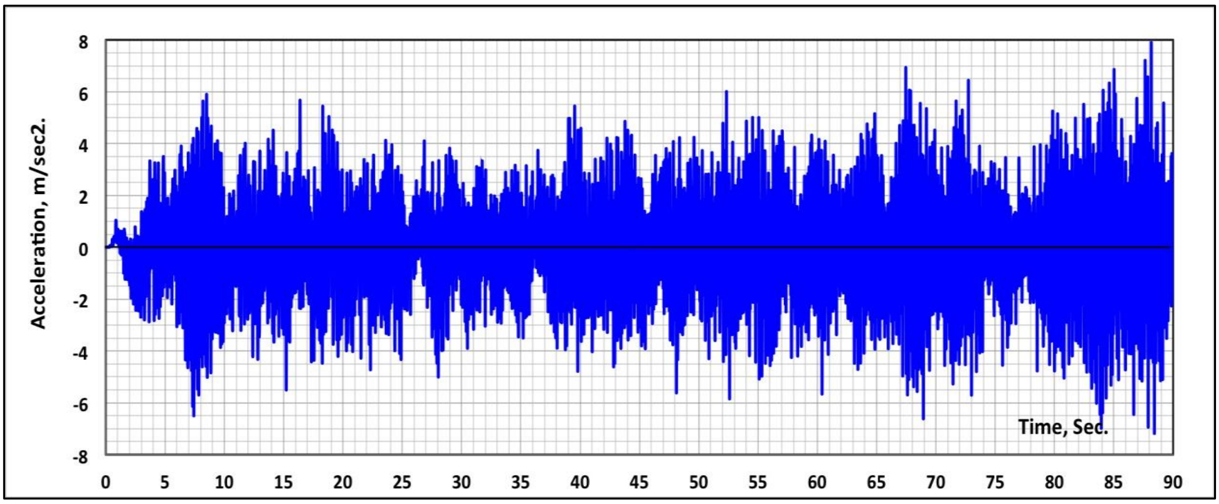


Fig. (11): Acceleration variation on the second span's mid-joint under the speed of 90km/hr.

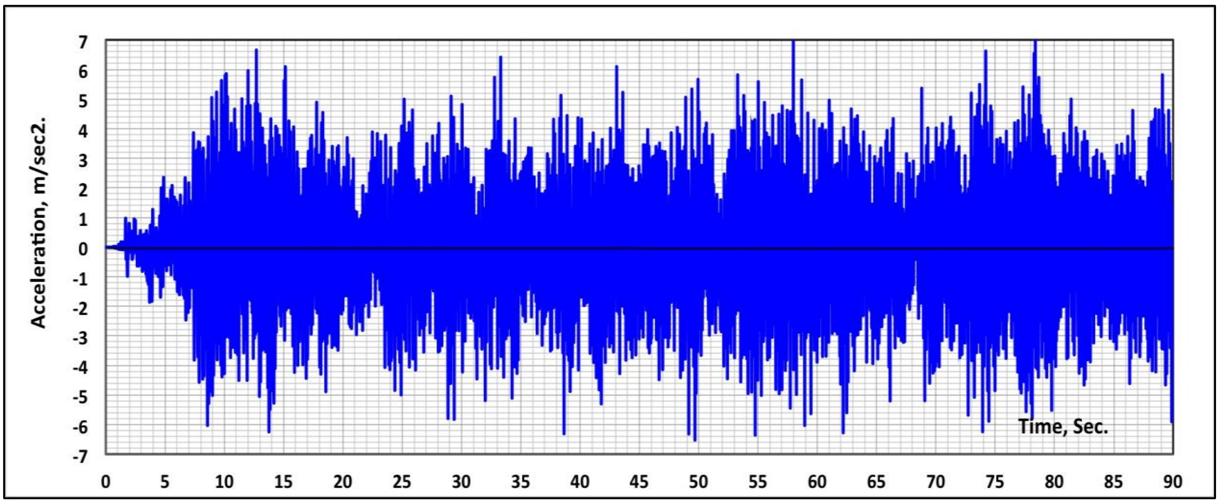


Fig. (15): Acceleration variation on the third span's mid-joint under the speed of 120km/hr.

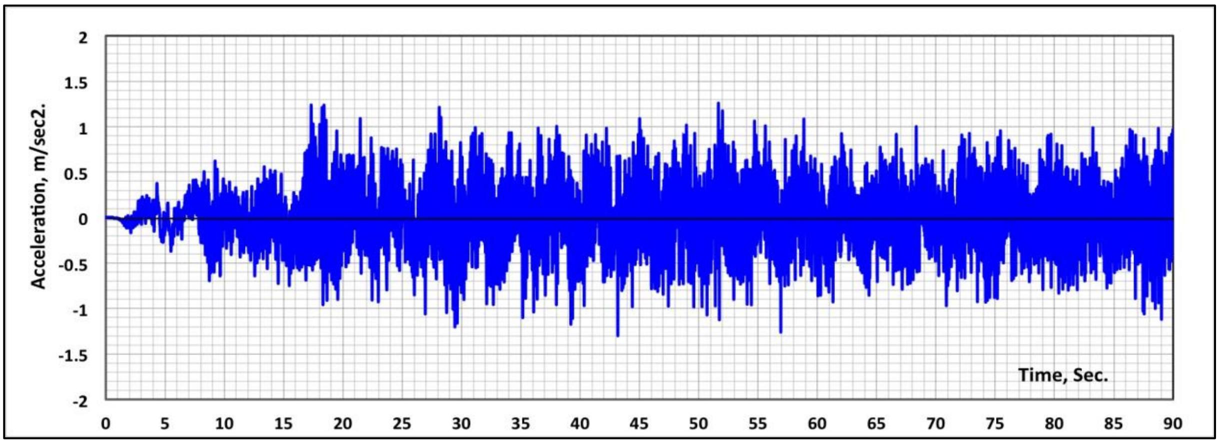


Fig. (16): Acceleration variation on the fourth span's mid-joint under the speed of 120km/hr.

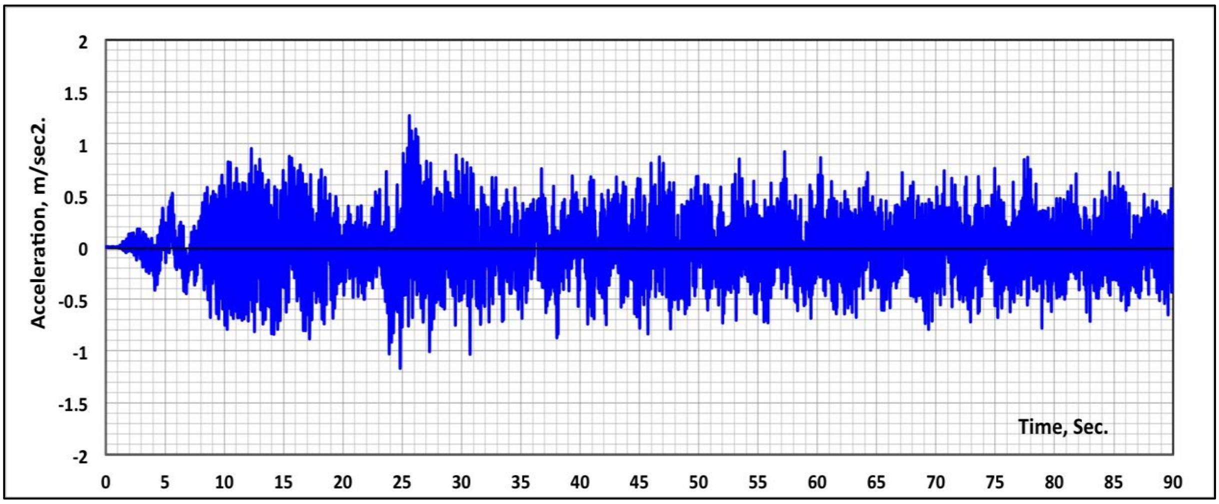


Fig. (14): Acceleration variation on the second span's mid-joint under the speed of 120km/hr.

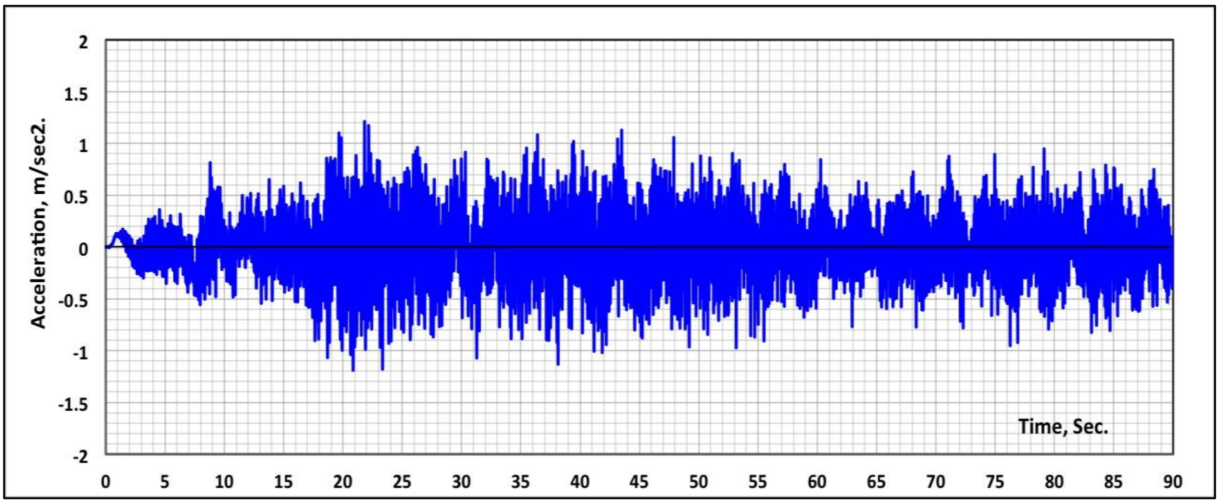


Fig. (18): Acceleration variation on the third span's mid-joint under the speed of 300km/hr.

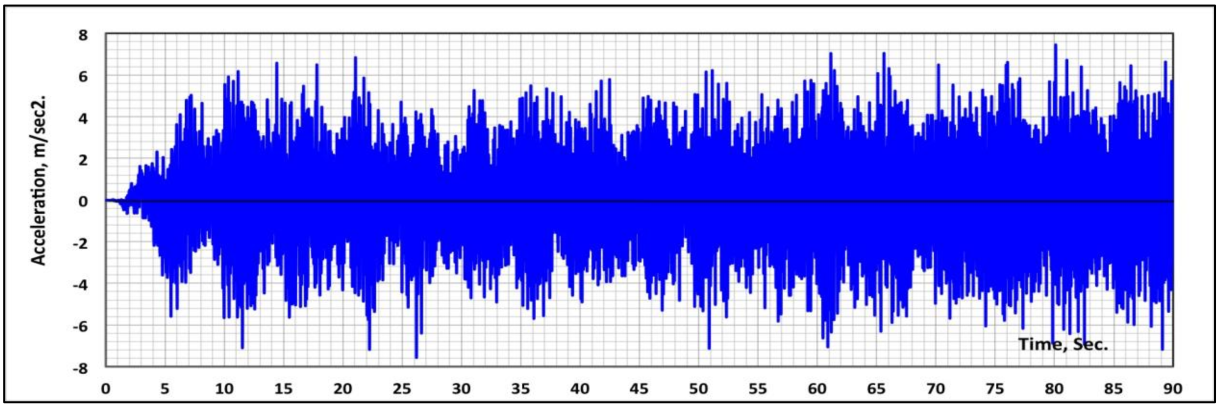


Fig. (19): Acceleration variation on the fourth span's mid-joint under the speed of 300km/hr.

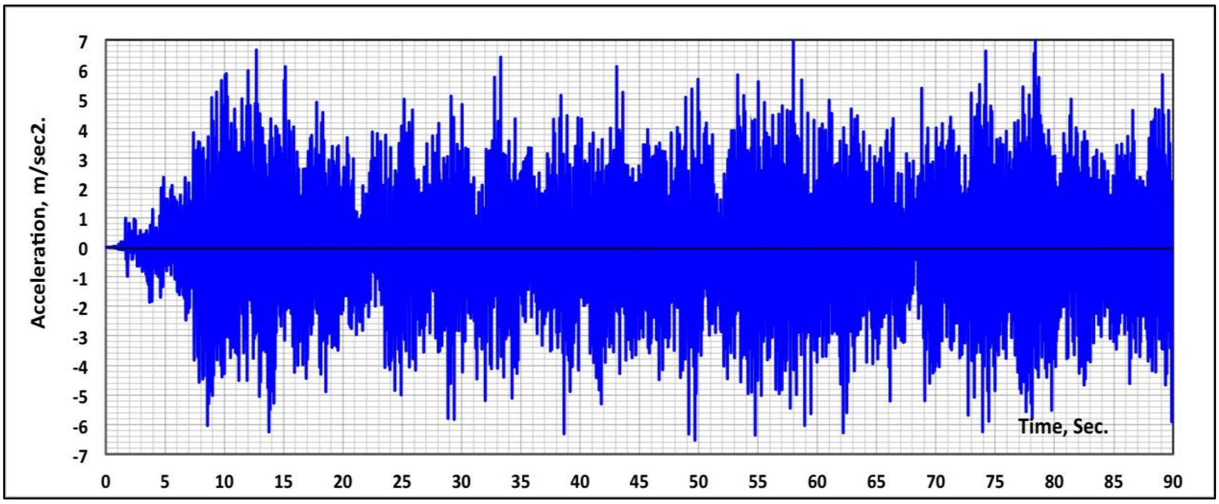


Fig. (17): Acceleration variation on the second span's mid-joint under the speed of 300km/hr.

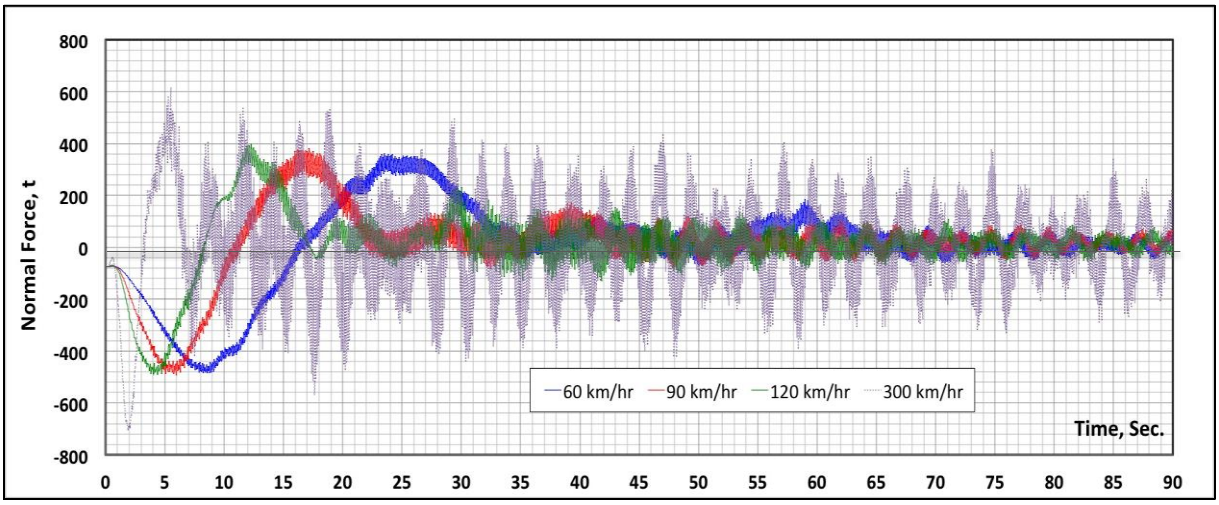


Fig. (21): Normal force variation on the third span's mid-joint under various load speeds

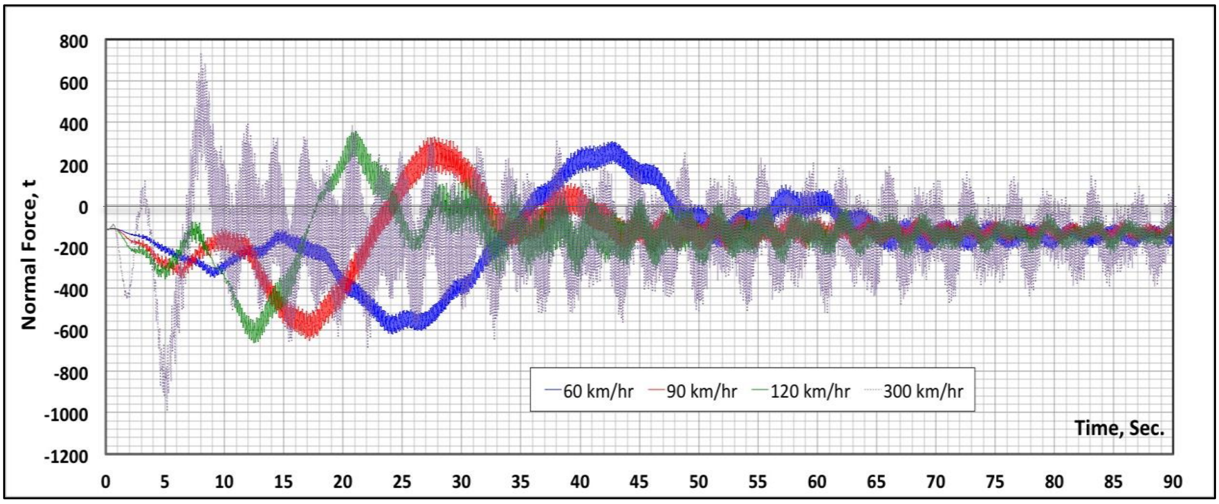


Fig. (20): Normal force variation on the second span's mid-joint under various load speeds

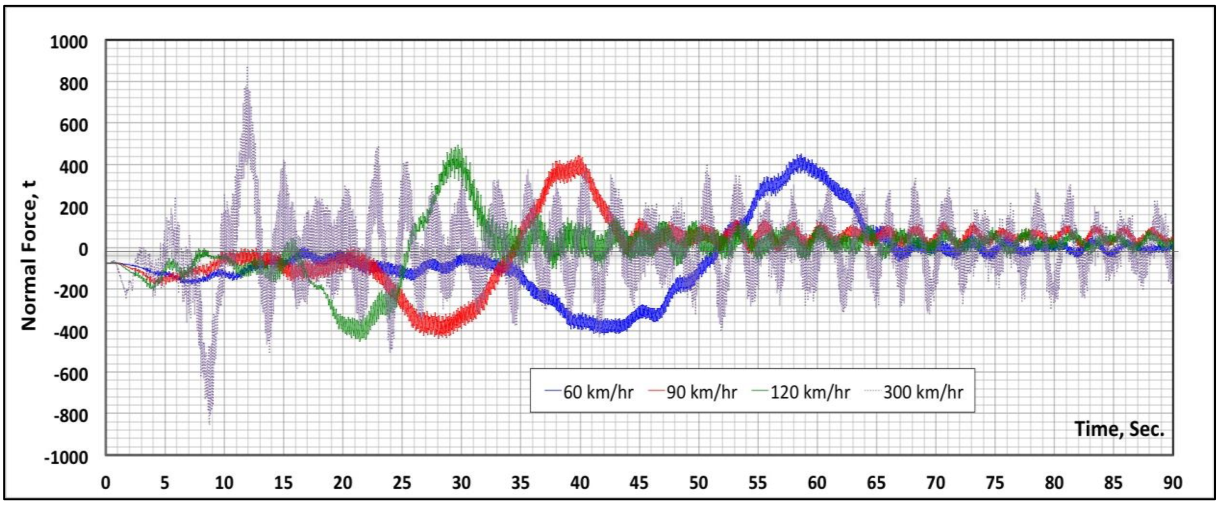


Fig. (24): Bending moment variation on the third span's mid-joint under various load speeds

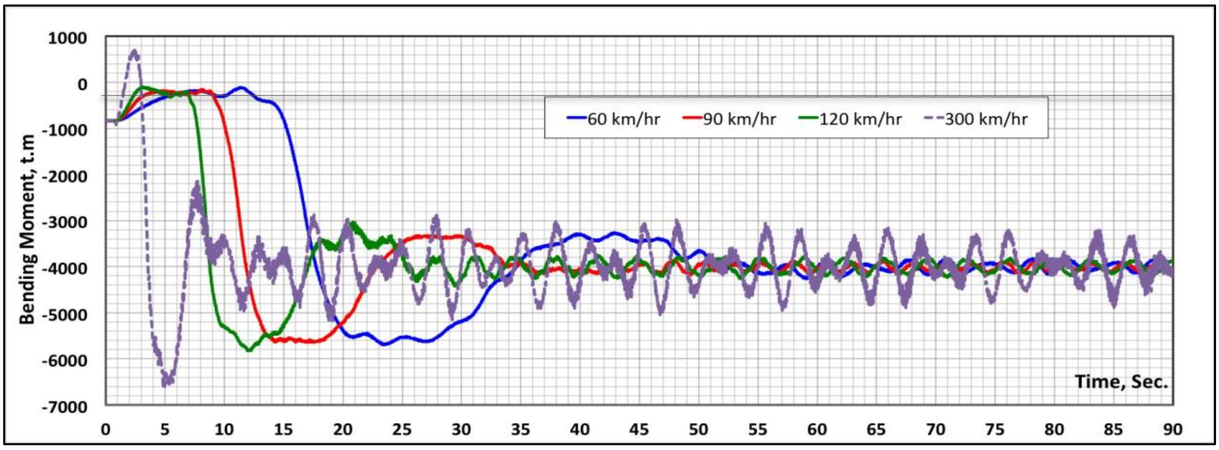


Fig. (22): Normal force variation on the fourth span's mid-joint under various load speeds

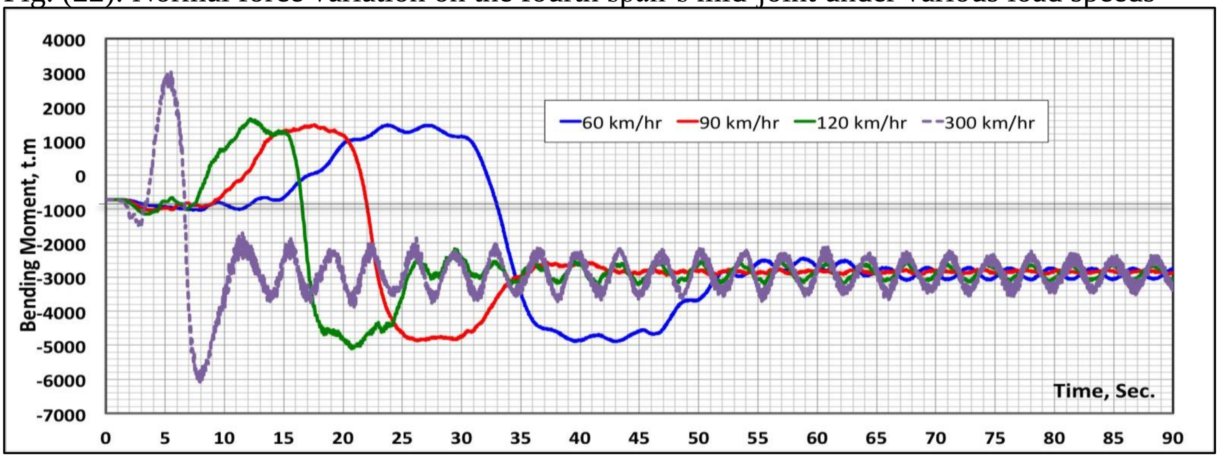


Fig. (23): Bending moment variation on the second span's mid-joint under various load speeds

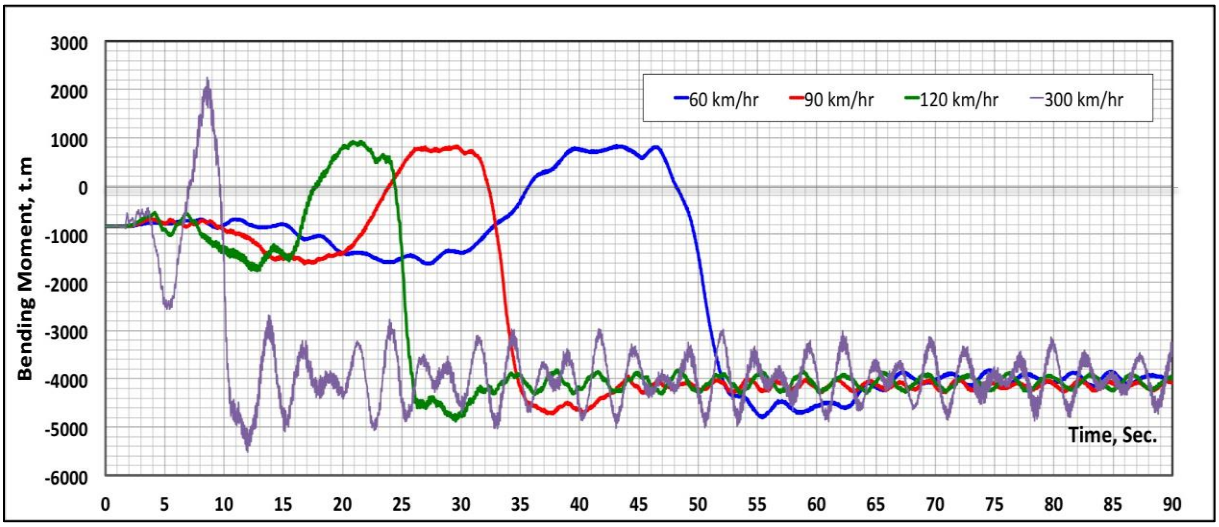


Fig. (25): Bending moment variation on the fourth span's mid-joint under various load speeds

5- Conclusions

1. The cycle of solution technique is the best choice for initial tension in cables.

2. All the dynamic responses increases with increasing the traffic load speed but with a shorter range, so it can be noticed that at high speeds, response could be safer than lower ones because of short period of effect.

3. Maximum acceleration is reached where zero velocity occurs.

6- References

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